A BEGINNING, ALBEIT NOT THE BEGINNING

IN FRITZ "SYNTHETIC APPROACH TO MARKOV KERNELS..."

"... THE DEFINING EQUATION OF MARGINALIZATION

$$f(y|a) := \sum_x f(x,y|a)$$

HEREBY ACQUIRES A COMPLETELY FORMAL MEANING VALID IN ANY MARKOV CATEGORY...

... DESPITE THE APPARENT UTILITY AND GENERALITY OF THIS NOTATION, WE WILL LEAVE A COMPLETE FORMALIZATION OF IT TO FUTURE WORK..."

As $(\mathrm{del}_X \otimes \mathrm{id}) \circ f$ for $f : A \to X \otimes Y$

DEFINITION DOTS MINE

THIS TALK DESCRIBES SUCH WORK

MODIFY SHULMAN'S

"PRACTICAL TYPE THEORY FOR SYMMETRIC MONOIDAL CATEGORIES"

TWICE OVER, INTO TYPE SYSTEMS FOR

- SEMICARTESIAN
- MARKOV CATEGORIES

MORE, WE WILL DO SO IN SUCH A WAY AS TO FIT WITH A LARGER GOAL

THE ACTUAL BEGINNING

A TYPE SYSTEM FOR PROBABILISTIC CONFORMATIONAL MOLECULAR PROGRAMMING

1

IN WHICH WE DEVELOP GLOBULAR COMPUTADS AS A THEORY OF ITERATIVE APPROXIMATION OF GLOBULAR ALGS.

Defn - Globes

.) $\mathbb{G}_n = \left\langle \bar{0} \overset{s}{\underset{t}{\rightrightarrows}} \bar{1} \rightrightarrows \cdots \rightrightarrows \bar{n} \;\middle|\; \begin{array}{l} s \circ t = s \circ s \\ t \circ t = t \circ s \end{array} \right\rangle$

..) $\partial \bar{0} = \emptyset$

...) $\partial \bar{k}_{\geq 1} = \overline{k-1} \sqcup_{\partial \overline{k-1}} \overline{k-1}$

....) $k = 0, 1, \ldots, n$

$$\partial \bar{k} \longrightarrow \bar{k}$$

THE "OBVIOUS" INCLUSIONS

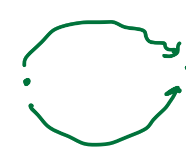
—) $\overline{n+1} = \bar{n}$

—:) $\partial \overline{n+1} \longrightarrow \overline{n+1}$

THE CODIAGONAL

$\partial \bar{0} = \emptyset$

$\partial \bar{1} = \bullet \quad \bullet$

$\partial \bar{2} =$ 

$\vdots$

DEFN – COMPUTADS

FIX T A FINITARY MONAD ON $\widehat{\mathbb{G}}_n$

i) $\mathrm{Comp}^T_{-1} = \bullet$

THE TERMINAL CATEGORY

ii)
$$\begin{tikzcd}
\mathrm{Comp}^T_{-1} \arrow[r, shift left=2, "P_{-1} = \ulcorner \emptyset \urcorner"] \arrow[r, phantom, "\perp"] & \widehat{\mathbb{G}}^T \arrow[l, shift left=2, "V_{-1} = !"]
\end{tikzcd}$$

CLASSIFIES THE INITIAL OBJECT

DEFINE

iii)
$$\begin{tikzcd}
\mathrm{Comp}^T_k \ \& \ \mathrm{Comp}^T_k \arrow[r, shift left=2, "P_k"] \arrow[r, phantom, "\perp"] & \widehat{\mathbb{G}}^T \arrow[l, shift left=2, "V_k"]
\end{tikzcd}$$

INDUCTIVELY

A K-1 COMP

•) $\mathrm{Comp}^T_K = \mathrm{Set} \downarrow \widehat{G}_n\left(\partial \bar{K}, U \circ P_{K-1}(-)\right)$

$$\begin{tikzcd}
 & \mathrm{Comp}^T_K \arrow[dl] \arrow[dr] & \\
\mathrm{Set} \arrow[rr, shift left] \arrow[rr, shift right] \arrow[dr, "\mathrm{Id}"'] & & \mathrm{Comp}^T_{K-1} \arrow[dl, "\widehat{G}_n\left(\partial \bar{K},\, U \circ P_{K-1}(-)\right)"] \\
 & \mathrm{Set} &
\end{tikzcd}$$

$\mathrm{Ob}(\mathrm{Comp}^T_K)$

$= \left\{ \coprod_{x \in X} \partial \bar{K} \xrightarrow{c} U \circ P_{K-1}(C) \right\}$

ABBREVIATED (X, C, c)

MAP FOR ATTACHING X-MANY K-GLOBES TO A T-ALG GENERATED BY A (K-1)-COMPUTAD C

DEFINE LEFT ADJOINTS

•) $P_k : \mathrm{Comp}_k^T \longrightarrow \widehat{\mathcal{B}}_n^T$

$\qquad (X, C, c) \longmapsto P_k(X, C, c)$

$\lrcorner$ in $\widehat{\mathcal{B}}_n^T$

$$\begin{tikzcd}
F^T\left(\coprod_X \partial k\right) \arrow[r] \arrow[d, "\bar{c}"'] & F^T\left(\coprod_X k\right) \arrow[d] \\
P_{k-1}(C) \arrow[r] & P_k(X, C, c) \arrow[ul, phantom, "\lrcorner"]
\end{tikzcd}$$

THE ALGEBRA GOTTEN BY ATTACHING X-MANY ALGEBRAS ON THE k-GLOBE TO $P_{k-1}(C)$ ALONG THE MAP c

DEFINE RIGHT ADJUNCTS

$$
\begin{aligned}
V_k : \widehat{\mathbb{G}}_n^{T} &\longrightarrow \mathrm{Comp}_k^{T} \\
(C,\mu) &\longmapsto \left(X_{(C,h)},\, V_{k-1}(C,\mu),\, x_{(C,h)}\right)
\end{aligned}
$$

Γ IN SET

$$
\begin{tikzcd}
X_{(C,h)} \arrow[r] \arrow[d, "x_{(C,h)}"'] \arrow[dr, phantom, "\ulcorner", very near start] & \widehat{\mathbb{G}}_n\left(\bar{K}, U(C,\mu)\right) \arrow[d, "(\partial K \to \bar{K})^*"] \\
\widehat{\mathbb{G}}_n\left(\partial \bar{K}, U \circ P_{k-1} \circ V_{k-1}(C,\mu)\right) \arrow[r, "\tilde{e}_*"] & \widehat{\mathbb{G}}_n\left(\partial \bar{K}, U(C,\mu)\right)
\end{tikzcd}
$$

THE ATTACHMENT OF A $\bar{K}$-GLOBE TO THE $(K-1)^{TH}$-APPROXIMATION OF C FOR EVERY CORRESPONDING K-GLOBE ATTACHED TO C

REMARK

RECALL

$$\left(\partial\overline{n+1} \longrightarrow \overline{n+1}\right) = \left(\overline{n} \sqcup_{\partial\overline{n}} \overline{n} \xrightarrow{\nabla} \overline{n}\right)$$

SO THE DEFINING PUSHOUT FOR P_{n+1}

$$\begin{tikzcd}
F^T\left(\coprod_X \overline{n} \sqcup_{\partial\overline{n}} \overline{n}\right) \arrow[r] \arrow[d] & F^T\left(\coprod_X \overline{n}\right) \arrow[d] \\
P_n(C) \arrow[r] \arrow[ur, phantom, "\lrcorner", very near end] & P_{n+1}(X, C, c)
\end{tikzcd}$$

IDENTIFIES n-CELLS OF $P_n(C)$ AS OPPOSED TO ATTACHING $n+1$-CELLS

REMARK

FOR ALL MONADS T

$$\mathrm{Comp}_0^T = \mathrm{Set}$$

SINCE

$$\cdot)\quad \partial\bar{o} = \phi \ \&\ U\circ P_{-1}(\cdot) = \phi$$

IT FOLLOWS THAT

$$\cdot\cdot)\quad \widehat{\mathfrak{R}}_u\left(\partial\bar{o}, U\circ P_{-1}\circ V_{-1}(\cdot)\right) = \{\cdot\}$$

WHENCE

$$\cdot\cdot\cdot)\quad \mathrm{Comp}_0^T = \mathrm{Set}\downarrow \widehat{\mathfrak{R}}_u\left(\partial\bar{o}, U\circ P_{-1}(\cdot)\right) = \mathrm{Set}$$

MORAL

I

THINK OF THE ~~DATA~~ OF COMPUTADS AS GENERATING CELLS IN DIMENSIONS $0, 1, \ldots, n$ GENERATING RELATIONS IN DIMENSION $n+1$

II

IF SKELETAL CONSTRUCTIONS

$$-1 \xrightarrow{\text{ATTACH}} 0 \xrightarrow{\text{ATTACH}} 1 \xrightarrow{\text{ATTACH}} 2 \longrightarrow \cdots$$

THEN COMPUTADS

$$-1 \xrightarrow{\text{ATTACH}} \xrightarrow{\text{GEN}} 0 \xrightarrow{\text{ATTACH}} \xrightarrow{\text{GEN}} 1 \xrightarrow{\text{ATTACH}} \xrightarrow{\text{GEN}} 2 \longrightarrow \cdots$$

III

THINK OF

$$\begin{tikzcd} \mathrm{Comp}_K^T \arrow[rr, bend left, "P_K"] \arrow[rr, phantom, "\Downarrow \varepsilon_K"] & & \widehat{\mathcal{G}}_n^{\,T} \arrow[ll, bend left, "V_K"] \end{tikzcd}$$

AS

- $P_K \circ V_K$ – THE K^{TH} COMPUTADIC APPROXIMATION FUNCTOR
- ε_K – THE COMPARISON

THEOREM - PERFECT APPROXIMATION (CSP, BATANIN)

ORIG.

FROM WHENCE I KNOW IT

FOR NICE ENOUGH MONADS T

$$\varepsilon_{n+1} : P_{n+1} \circ V_{n+1} \Longrightarrow \mathrm{Id}$$

IS A NATURAL EQUIVALENCE

OF T-ALGS,
I.E.
EQ OF CATS
2-EQ OF 2-CATS
⋮

THEOREM - COMPUTADS ARE OFTEN PRESHEAVES (BATANIN)

FOR NICE ENOUGH MONADS T

&

NICE ENOUGH VALUES OF K

$$\mathrm{COMP}^T_K = \widehat{B^T_K}$$

FOR SOME SMALL CATEGORY B^T_K

Corollary

For monads T for

- symmetric monoidal categories
- semicartesian monoidal categories
- Markov categories

and $k = 0, 1, 2$

$$\mathrm{Comp}_k^T \xrightarrow{\sim} \widehat{\mathcal{B}}_k^T$$

and

$$\varepsilon_2 : P_2 \circ V_2 \Longrightarrow \mathrm{Id}$$

is a natural equivalence

2

IN WHICH WE ELABORATE THE CATEGORIES OF 0, 1, AND 2 COMPUTADS FOR CATEGORIES SYMMETRIC MONOIDAL, SEMICARTESIAN, AND MARKOV

0-computads for SMCs

Let T be the monad for SMCs.

$$\mathrm{Set} = \mathrm{Comp}_0^T \underset{V_0 = \mathrm{Ob}}{\overset{P_0 = \Sigma(-)}{\rightleftarrows}} \mathrm{SMC}$$

Always true since

$\mathcal{N}$ from MMSS

$$\{\cdot\} \longmapsto \Sigma$$

$$\mathrm{Ob}(\Sigma) = \{\langle n\rangle = \{1,2,\dots,n\}\}$$

$$\Sigma(\langle n\rangle, \langle m\rangle) = \mathrm{Set}_B^{op}(\langle n\rangle, \langle m\rangle)$$

$$X \longmapsto \Sigma_X$$

$$\begin{tikzcd}
 & X \downarrow \Sigma \arrow[dl] \arrow[dr] & \\
X \arrow[dr, "{[x]}"'] \arrow[rr, phantom, "\Longrightarrow"] & & \Sigma \arrow[dl] \\
 & \mathrm{Set}^{op} &
\end{tikzcd}$$

$$\mathrm{Ob}(\Sigma_X) = \mathrm{List}(X)$$

$$\Sigma_X((x_1, x_2, \dots, x_n), (y_1, y_2, \dots, y_m))$$

$$\parallel$$

$$\{\sigma \in \mathrm{Set}_B^{op}(\langle n\rangle, \langle m\rangle) \mid \forall j.\, x_{\sigma(j)} = y_j\}$$

1-COMPUTADS FOR SMCs

SINCE

$$P_0 = \Sigma_{(-)} \quad \& \quad \partial T = \bullet \sqcup \bullet$$

IT FOLLOWS THAT

$\bullet$) $\widehat{\mathcal{G}}_1(\partial T, V \circ P_0(\ )) : \mathrm{SET} \longrightarrow \mathrm{SET}$

$$X \longmapsto \mathrm{List}(X)^2$$

SO A 1-COMPUTAD IS

$\bullet\bullet$) $\coprod_{x \in X} \partial T \xrightarrow{(s,t)} \mathrm{List}(C)$

THE CATEGORY OF SUCH IS A PRESHEAF TOPOS

THE CATEGORY H_1

LET H_1 BE THE CATEGORY WITH

$$\mathrm{Ob}(H_1) = \{\uparrow\} \cup \left\{ \begin{array}{c} \overbrace{\uparrow \cdots \uparrow}^{n\text{-MANY}} \\ \boxed{\frac{n}{m}} \\ \underbrace{\uparrow \cdots \uparrow}_{m\text{-MANY}} \end{array} \right\}_{n,m \in \mathbb{N}}$$

PRESHEAVES ON H_1 ARE OFTEN CALLED HYPERGRAPHS

AND MORPHISMS FREELY GENERATED BY THE SETS

$$H_1\left(\uparrow, \begin{array}{c} \uparrow \cdots \uparrow \\ \boxed{\frac{n}{m}} \\ \uparrow \cdots \uparrow \end{array}\right) = \{ in_i \mid i \in \langle m \rangle \} \sqcup \{ out_j \mid j \in \langle n \rangle \}$$

THE EQUIVALENCE PUTS IN CORRESPONDENCE

$$\frac{X : H_1^{op} \longrightarrow \mathrm{Set}}{\displaystyle\coprod_{n,m \in \mathbb{N}} \coprod_{x \in X(\boxed{\frac{n}{m}})} 1 \xrightarrow{((in_i),(out_j))} \mathrm{List}(X(\uparrow))}$$

THE RIGHT ADJUNCT V_1

THE RIGHT ADJUNCT

$$\begin{tikzcd} \mathrm{Comp}_1^T & SMC \arrow[l, "V_1"'] \\ V_1(C) & C \arrow[l, mapsto] \end{tikzcd}$$

DEFINED BY LIMITS BY ~~SET~~ PULLBACKS

$$\begin{tikzcd} X_{(c,1)} \arrow[r] \arrow[d, "x_{(c,1)}"'] & Q(T, U(c)) \arrow[d] \\ Q(\partial T, U \circ P_0 \circ V_0(c)) \arrow[r] & Q_1(\partial T, U(c)) \end{tikzcd}$$

NODES ARE LISTS OF OBJECTS IN C W/ AN EDGE FOR EVERY WAY TO WRITE THE SOURCE & TARGET OF IT

($E.G.\ A = B \otimes C = D \otimes E \otimes F$)

THE LEFT ADJUNCT P_1 - I

THE LEFT ADJUNCT

$$\mathrm{Comp}^T_1 \xrightarrow{P_1} \mathrm{SMC}$$

$$X \longmapsto \text{THE SMC GEN BY } \Sigma_{X(\uparrow)} \ \& \ \coprod X\left(\boxed{\tfrac{n}{m}}\right)$$

WAY TOO VAGUE FOR COMPUTATION

THE LEFT ADJUNCT P_1 – II

LEMMA (SHULMAN)

For any $X : H_1^{op} \to \mathrm{Set}$ of Comp^T_1, every morphism of $P_1(X)$ is of the form

AWD if meets those criteria

From $X\left(\boxed{\frac{n \geq 1}{m \geq 1}}\right)$ functions

From $X\left(\boxed{\frac{0}{m \geq 1}}\right)$ scalar valued funct.

From $X\left(\boxed{\frac{n \geq 1}{0}}\right)$ constant gen

From $X\left(\boxed{\frac{0}{0}}\right)$ scalar gen.

More, the form can be made canonical by adding conditions on

- ~~Apply~~ applyin □'s to |...|
- order preservation properties of

Call such AWD's

2-COMPUTADS FOR SMCs

COROLLARY

THE CATEGORY Comp_2^T IS A PRESHEAF TOPOS. ~~INDEED~~ LET H_2 BE THE CAT

$$\mathrm{Ob}(H_2) = \{\uparrow\} \cup \left\{\boxed{\frac{n}{m}}\right\} \cup \left\{\boxed{\frac{\tau}{\rho}}\right\}$$

$$\mathrm{Mor}(H_2) = \text{INCLUSIONS OF WIRES \& WIRING DIAGRAM COMPONENTS}$$

GENERIC AWDs

REMARK

PROPs ARE PRECISELY THOSE SMCs OF THE FORM

$P_0(X)$,

$P_1((X, C, c))$, or

$P_2((X, C, c))$

I.E.

PROPs = COMPUTADIC SMCs

CATEGORIES SEMICARTESIAN & MARKOV

DEFN

A SEMICARTESIAN CATEGORY IS SYMMETRIC MONOIDAL CATEGORY IN WHICH THE UNIT IS TERMINAL. DENOTE THE ASSOCIATED 1-GLOBULAR MONAD BY S

DEFN

GIVEN A SEMICARTESIAN CATEGORY $(\mathcal{A}, \otimes, \bullet)$ A MARKOV CATEGORY STRUCTURE ON $\mathcal{A}$ IS AN $\mathrm{Ob}(\mathcal{A})$-INDEXED FAMILY OF CO-ASSOCIATIVE CO-MONOIDS

$$\left\{ \left(\mathrm{Copy}_A : A \to A \otimes A ,\ \mathrm{Del}_A : A \to \bullet \right) \right\}_{A \in \mathrm{Ob}(\mathcal{A})}$$

WHICH MOREOVER PRESERVES $\otimes$. DENOTE THE ASSOCIATED 1-GLOBULAR MONAD BY M

0-COMPUTADS FOR SCMCS   MARKOV CATS I

THE CATEGORY

$\Sigma \xrightarrow{\sim} \mathrm{FinSet}^{op}_{B}$

IS NOT SEMICARTESIAN B/C MISSING THE CANONICAL MAPS TO <0> & THE MAPS THOSE GENERATE

$\longleftrightarrow$

THE CATEGORY

$\Sigma^{op} \xrightarrow{\sim} \mathrm{FinSet}_{B}$

IS NOT SEMI-CO-CARTESIAN B/C MISSING THE CANONICAL MAPS FROM <0> & THE MAPS THOSE GENERATE

TO MAKE Σ SEMICARTESIAN WE [MONO / BIJECTION]

DEFINITION

$K :=$ FULL SUBCAT OF $\mathrm{Set}^{op}_{\mathrm{Mono}}$ ON OBJECTS $\{ \langle n \rangle \}_{n \in \mathbb{N}}$

SET OF

O-COMPUTADS FOR SCMCS & MARKOV CATEGORIES II

THE CATEGORY

$\mathbb{K} :\xrightarrow{\sim} \mathrm{FinSet}_M^{op}$

IS NOT MARKOV

NEED TO ADD DUPLICATING MAPS

$\longleftrightarrow$

THE CATEGORY

$\mathbb{K}^{op} :\xrightarrow{\sim} \mathrm{FinSet}_M$

IS NOT CO-MARKOV

NEED TO ADD RAMIFIED MAPS

TO MAKE K MARKOV WE $\left[\text{ALL} / \text{MONO}\right]$

DEFINITION

$$M := \begin{array}{l}\text{FULL SUBCAT OF } \mathrm{Set}^{op} \\ \text{ON SET OF OBJECTS } \left\{ \langle n \rangle \right\}_{n \in \mathbb{N}}\end{array}$$

DEFN

LET

$$K_{(-)} : \mathrm{Comp}_0^{S} : \mathrm{Set} \longrightarrow \mathrm{SCMC}$$

$$X \longmapsto K_X$$

$$\begin{tikzcd} & K_X \arrow[dl] \arrow[dr] & \\ X \arrow[dr] \arrow[rr, phantom, "\Rightarrow"] & & K \arrow[dl] \\ & \mathrm{Set}^{op} & \end{tikzcd}$$

$$\begin{cases} \mathrm{Ob}(K_X) = \mathrm{List}(X) \\ K_X\big((x_i)_{\langle n\rangle}, (y_j)_{\langle m\rangle}\big) = \left\{ \sigma \in K(\langle n\rangle, \langle m\rangle) \;\middle|\; \forall j \in \langle m\rangle .\, x_{\sigma(j)} = y_j \right\} \end{cases}$$

$$M_{(-)} : \mathrm{Comp}_0^{M} : \mathrm{Set} \longrightarrow \mathrm{Markov\ Cat}$$

$$X \longmapsto M_X$$

$$\begin{tikzcd} & M_X \arrow[dl] \arrow[dr] & \\ X \arrow[dr] \arrow[rr, phantom, "\Rightarrow"] & & M \arrow[dl] \\ & \mathrm{Set}^{op} & \end{tikzcd}$$

$$\begin{cases} \mathrm{Ob}(M_X) \\ M_X\big((x_i)_{\langle n\rangle}, (y_j)_{\langle m\rangle}\big) = \left\{ \sigma \in M(\langle n\rangle, \langle m\rangle) \;\middle|\; \forall j \in \langle m\rangle .\, X_{\sigma(j)} = y_j \right\} \end{cases}$$

0-COMPUTADS FOR SCMCS & MARKOV CATS II

$$
\begin{tikzcd}[column sep=huge, row sep=large]
\mathrm{Comp}_0^T = \mathrm{Set} \arrow[r, shift left=1.5ex, "P_0^T = \Sigma(-)"] \arrow[r, phantom, "\perp"] & \mathrm{SMC} \arrow[l, shift left=1.5ex, "V_0 = \mathrm{Ob}"] \\
\mathrm{Comp}_0^S = \mathrm{Set} \arrow[u] \arrow[r, shift left=1.5ex, "P_0^S = K(-)"] \arrow[r, phantom, "\perp"] \arrow[ur, phantom, "\mathrm{LA}\ \Downarrow \qquad\qquad \mathrm{RA}\ \circlearrowleft"] & \mathrm{SCMC} \arrow[u] \arrow[l, shift left=1.5ex, "V_0 = \mathrm{Ob}"] \\
\mathrm{Comp}_0^M = \mathrm{Set} \arrow[u] \arrow[r, shift left=1.5ex, "P_0^M = M(-)"] \arrow[ur, phantom, "\mathrm{LA}\ \Downarrow \qquad\qquad \mathrm{RA}\ \circlearrowleft"] & \mathrm{Markov} \arrow[u] \arrow[l, shift left=1.5ex]
\end{tikzcd}
$$

COMPOSE MORPHISMS OF ADJUNCTIONS

1- COMPUTADS FOR SCMCS & MARKOV CATEGORIES

WHILE

$$\underbrace{U \circ P_0^T(X)}_{\Sigma_X} \subsetneq \underbrace{U \circ P_0^S(X)}_{K_X} \subsetneq \underbrace{U \circ P_0^M(X)}_{M_X}$$

WE HAVE

$$\widehat{G}(\partial T, U \circ P_0^T(X)) \simeq \widehat{G}(\partial T, U \circ P_0^S(X)) \xrightarrow{\sim} \widehat{G}(\partial T, U \circ P_0^M(X))$$

SO IT FOLLOWS THAT

$$\mathrm{Comp}_1^T \xrightarrow{\sim} \mathrm{Comp}_1^S \xrightarrow{\sim} \mathrm{Comp}_1^M$$

THE RIGHT ADJOINTS V_1^S & V_1^M

MORE SEE THAT THE SET CO-SPANS

DEFINE FUNCTORS

$$
\begin{tikzcd}
V_1^A \arrow[r] \arrow[d] \arrow[dr, phantom, "\lrcorner", very near start] & \hat{G}_1(T, U(C)) \arrow[d] \\
\hat{G}_1(\partial T, \underbrace{U \circ P_0^A \circ V_0^A(C)}_{\text{B/C SAME PAIRS OF LISTS}}) \arrow[r] & \hat{G}_1(\partial T, U(C))
\end{tikzcd}
$$

ARE THE SAME FOR $A = T, S, M$. THUS

$$
\begin{tikzcd}
\mathrm{Comp}_1^T & & \mathrm{SMC} \arrow[ll, "V_1^T"'] \\
\mathrm{Comp}_1^S \arrow[u] & \circlearrowright & \mathrm{SCMC} \arrow[ll, "V_1^S"'] \arrow[u] \\
\mathrm{Comp}_1^M \arrow[u] & \circlearrowright & \mathrm{MARKOV} \arrow[ll, "V_1^M"'] \arrow[u]
\end{tikzcd}
$$

THE LEFT ADJUNCTS P_i^S & P_i^M - I

RECALL THAT

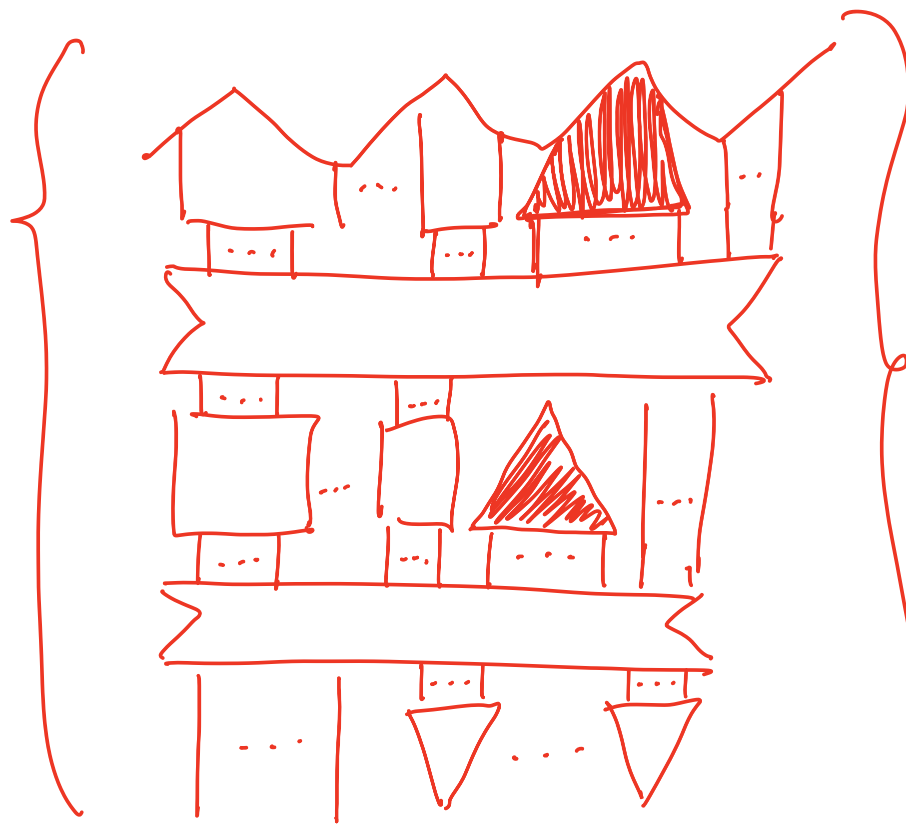
$$\mathrm{Mor}(P_i^T(X)) \xrightarrow{\sim}$$

ORDER OF SCALARS BREAKS CANONICITY

THE LEFT ADJUNCTS P_i^S & P_i^M – II

IN A SEMICARTESIAN CATEGORY

- THERE ARE NO NON IDENTITY $\bigcirc$ ← SCALARS
- THE ONLY $\triangle = \blacktriangle$ ← CANONICAL MAP TO THE TERMINAL UNIT

$\mathrm{Mor}(P_i(X)) \xrightarrow{\sim}$ 

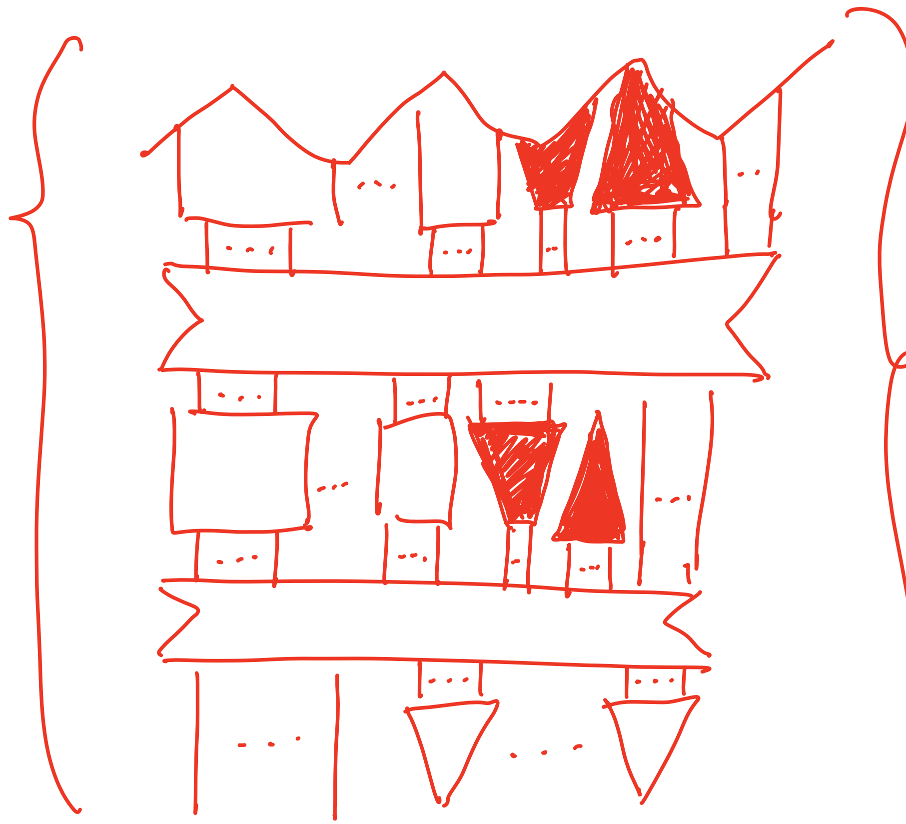
NO MORE ~

THE LEFT ADJUNCTS P_i^S & P_i^M – III

IN A MARKOV CAT

- THERE ARE NO NON IDENTITY ○ — SCALARS
- THE ONLY △ = ▲ — CANONICAL MAP TO THE TERMINAL UNIT
- THERE ARE DUPLICATIONS ▼

$$Mor(P_i^T(X)) \cong$$



NO MORE ~

2 – COMPS

OMITTED

FOR TIME

3

IN WHICH THEORIES OF TUPLES BEGET THEORIES OF "SETS-WITH-ELEMENTS".

PTT AND SEMICARTESIAN AND MARKOV VARIANTS

PRACTICAL TYPE THEORY

SHULMAN'S PTT IS A $\boxed{\text{SETS-WITH-ELTS}}$ FLAVORED TYPE THEORY FOR SYMMETRIC MONOIDAL CATEGORIES

$$\mathrm{SMC} \underset{\text{SYNTACTIC CATEGORY}}{\overset{\text{THEORY OF}}{\rightleftarrows}} \mathrm{PTT}$$

OBJECTS $\longleftrightarrow$ TYPES

MORPHISMS $\longleftrightarrow$ DER. TYPING JUDGEMENTS

COMMUTING POLYGONS $\longleftrightarrow$ DER. EQUALITY JUDGEMENTS

$$\frac{(f\otimes g)\circ\sigma : B\otimes A \longrightarrow A\otimes B \longrightarrow C\otimes D}{(b,a):(B,A) \vdash (f(a),g(b)):(C,D)}$$

A SOCRATIC PUPPET SHOW?

BUT YOU OBJECT:

EVEN IN THE MOST UBIQUITOUS CONCRETE SMC: E.G. $\mathbb{R}$-VECT, WHILE MONOIDAL PRODUCTS OF OBJECTS CERTAINLY HAVE ELEMENTS, THOSE ELEMENTS ARE NOT, IN GENERAL, MERE TUPLES, ~~THAT IS~~ MERE SIMPLE TENSORS.

THE SYNTHETIC MATHEMATICIAN RESPONDS

WE DON'T NEED ELEMENTS TO "BE" TUPLES

WE NEED TUPLES TO CARRY ENOUGH INFORMATION SO THAT PERMISSABLE INFERENCE AND MANIPULATION OF THEM CORRESPONDS TO INFERENCE AND MANIPULATION IN/OF THE CATEGORY

PUTTING WORDS IN YOUR MOUTH

I

SETS-WITH-ELEMENTS?

WHAT MAKES A TYPING JUDGEMENT

$(x,y) : A \times B \longmapsfrom (f(g(x)), h(y)) : C \times D$

LOOK AND FEEL LIKE A MORPHISM OF SETS?

SYMMETRY BETWEEN CONTEXTS

AND TYPING CONSEQUENTS

LIST OF VARIABLES
LIST OF TYPES

LIST OF TERMS
LIST OF TYPES

THE FAINT REEK OF OUR 0. COMPUTADS AESTHETICS OF TUPLES

DERIVATION AS GENERATION

DERIVATION OF THE JUDGEMENTS OF A TYPE SYSTEM $\xrightarrow[\text{FLAVOR OF}]{\text{PARTICULAR}}$ GENERATION OF AN ALGEBRA

FOR THESE TWO WE'LL ACTUALLY CUT TO THE CHASE

WRITE

$$\mathrm{Comp}^{T,S,M}_{0,1,2} \xrightarrow{P^{T,S,M}_{0,1,2}} \widehat{C}^{T,S,M}_{1}$$

$$\mathbb{X} \longmapsto \mathrm{SynCat}\left(PTT^{0,1,2}_{\mathbb{X}},\ SCPTT^{0,1,2}_{\mathbb{X}},\ MPTT^{0,1,2}_{\mathbb{X}}\right)$$

WHERE

SYN CAT (TYPE SYSTEM) { HAS CONTEXTS/~ AS OBJECTS

HAS DER TYPING JUDGEMENTS/~ AS MORPHISMS

VARIABLE NAMES

VARIABLE NAMES & DERIVABLE EQ. JUDGEMENTS

A REMARK ABOUT DETAIL

REMARK

TIME IS SHORT (AM I NOT ALREADY OVER TIME?)

TREAT TERM FORMATION RULES VERY INFORMALLY

OMIT SOME TECHNICAL DETAILS

"ACTIVENESS" MAY BE SKIPPED OVER FOR EXPOSITORY SIMPLICITY

SAVE THESE KINDS OF QUESTIONS UNTIL LATER

PTT° I - DESIDERATA FOR PTT°

TO DEFINE $PTT^{\circ}_{(\_)}$ WE NEED TO DEFINE

.) TYPES & CONTEXTS

..) TERM FORMATION RULES

...) RULES FOR THE TYPING JUDGEMENT

....) RULES FOR THE EQUALITY JUDGEMENT

S.T. FOR EACH $X \in$ Set / 0-COMPUTAD

.) TYPES, CONTEXTS$/\sim$ OF $PTT^{\circ}_{X} \xrightarrow{\sim} \mathrm{List}(X)$

NAMES OF VARIABLES

..) D. TYPING JUDGEMENTS$/\sim$ D. EQUALITY JUDGEMENTS $\xrightarrow{\sim} \mathrm{Mor}(\Sigma_X)$

PERMUTATIONS OF LISTS

PTT° II - Types & Contexts of PTT°_{X}

.) Types of $PTT^{\circ}_{X_0} :=$ List(X)

- $(A_1, A_2, \ldots, A_n)$
- $\vec{A}$
- Δ

..) Contexts of $PTT^{\circ}_{X_0} :=$ List$\left(\left\{ (x:A) \,\middle|\, \begin{array}{l} x \text{ var. symbol} \\ A \in X \end{array} \right\}\right)$

- $(x_1 : A_1, x_2 : A_2, \ldots, x_n : A_n)$
- $\vec{x} : \vec{A}$
- Γ

PTT° III – TERMS & TYPING JUDGEMENTS FOR PTT°_{X}

.) $$\frac{(x : A) \in \Gamma}{\Gamma \vdash x \ \text{TERM}}$$

ALLOWS US TO USE VARIABLES AS TERMS

..) $$\frac{\sigma : (\vec{A}) \xrightarrow{\sim} \Delta}{\vec{x} : \vec{A} \leftarrow \sigma(\vec{x}) : \Delta}$$

A PERMUTATION OF LISTS

SHORTHAND FOR REORGANIZING THE VARIABLES OF THE CONTEXT $\vec{x} : \vec{A}$ ACCORDING TO σ

LEMMA

DER. TYPING JUDGEMENTS$/\sim$

NAMES OF VARIABLES

$$\downarrow \wr$$

$$\mathrm{Mor}(\Sigma_X)$$

MEANS WE DON'T NEED EQ. JUDGEMENTS FOR THE 0-FRAGMENT

PTT' I – DESIDERATA FOR PTT'

GIVEN A 1-COMPUTAD $\mathbb{X} = (X_1, X_0, (s,t))$

$\text{Types \& Contexts}(PTT'_{\mathbb{X}}) := \text{Types \& Contexts}(PTT^0_{X_0})$

EXPEND THE RULES FOR

- .) TERM FORMATION
- ..) TYPING JUDGEMENT
- ...) EQUALITY JUDGEMENT

SO THAT

$$\left. \text{DERIVABLE TYPING JUDGEMENTS} \middle/ \begin{array}{l}\text{VARIABLE NAMES} \\ \sim \text{DERIVABLE EQ. JUDGEMENTS}\end{array} \right. \xrightarrow{\sim} \mathrm{Mor}(P_1^T(\mathbb{X}))$$

RECALL THESE ARE JUST AWDs

PTT' II - TERM FORMATION RULES

ADD TERM FORMATION RULES SO THAT

GIVES RISE TO

$f \in X\left(\boxed{\frac{n=0,1}{m \geq 1}}\right) \rightsquigarrow \underbrace{f(x_1, x_2, \dots, x_m)}_{\text{PREEXISTING TERMS}}$ TERM

$f \in X\left(\boxed{\frac{n \geq 2}{m \geq 1}}\right) \rightsquigarrow f_{(i)}(x_1, x_2, \dots, x_m)$ TERM

$i \in \langle n \rangle$

$f \in X\left(\boxed{\frac{n=0}{m=0}}\right) \rightsquigarrow f$ TERM

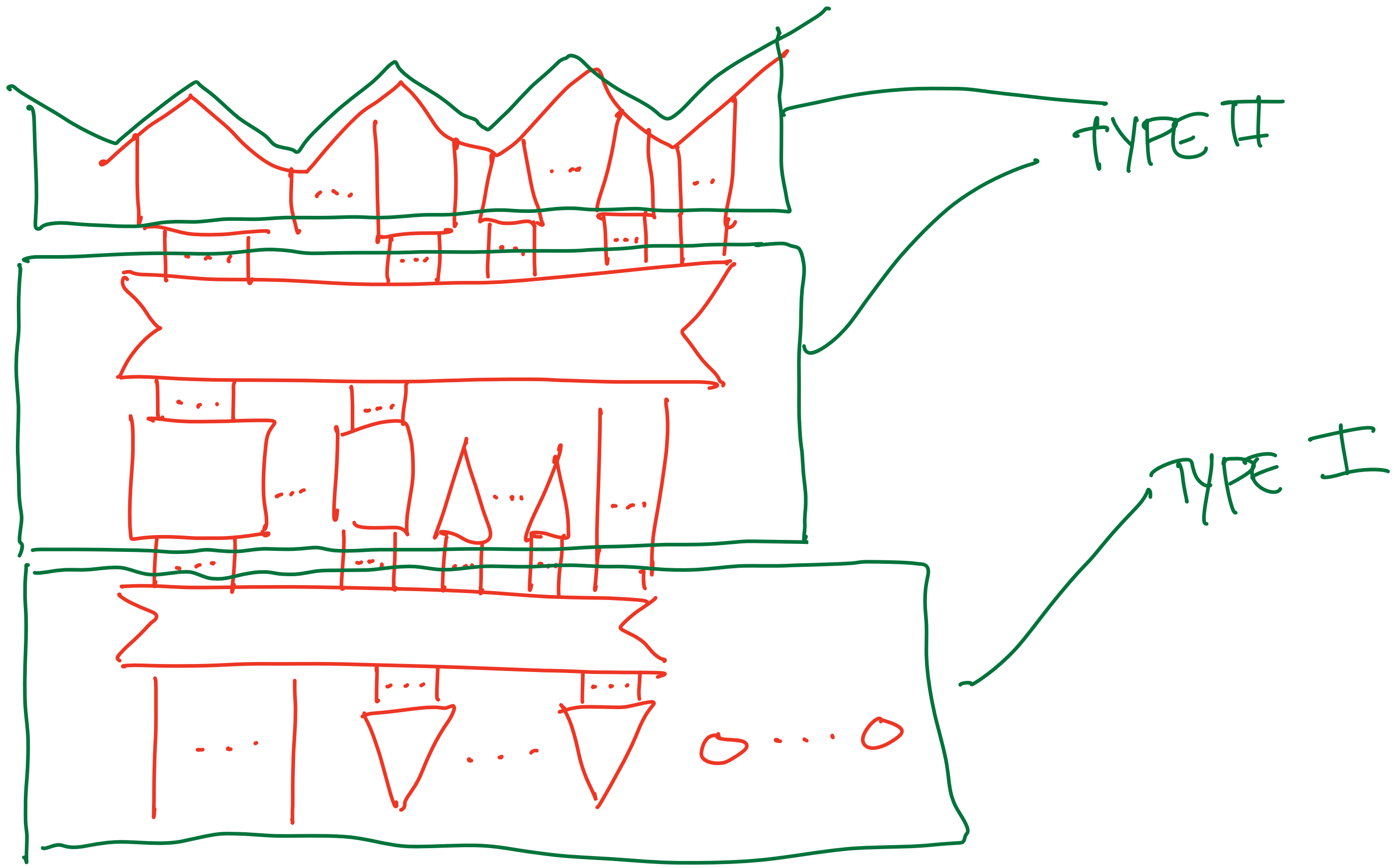
$f \in X\left(\boxed{\frac{n=1}{m=0}}\right) \longrightarrow f^{a}_{\text{TERM}}$

A LABEL IN AN ALPHABET (SYNTACTIC SUGAR)

$f \in X\left(\boxed{\frac{n \geq 2}{m=0}}\right) \rightsquigarrow f^{a}_{(i)}$ TERM

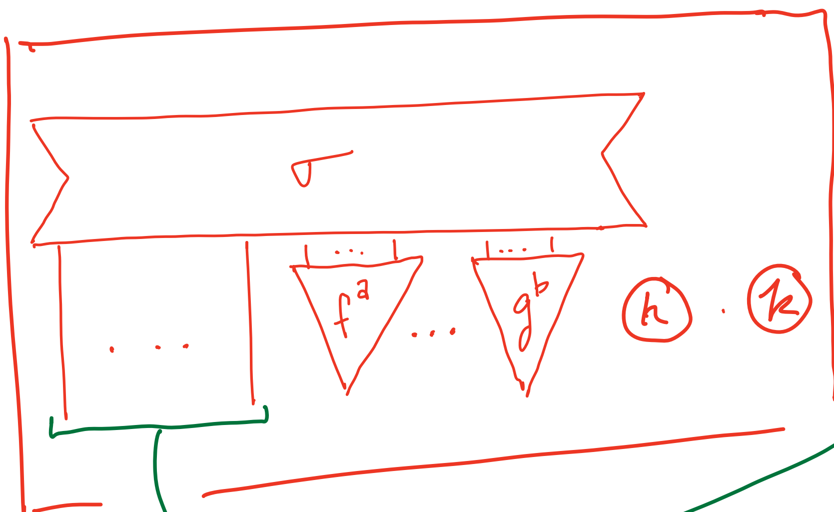
$\boxed{\text{N.B.} \quad x_i \leftarrow \text{THAT'S FOR US} \qquad x_{(i)} \leftarrow \text{THAT'S IN THE SYNTAX}}$

PTT' III.I - TYPING JUDGEMENT RULES



PTT' III.II - TYPE I RULE

ENCODED BY

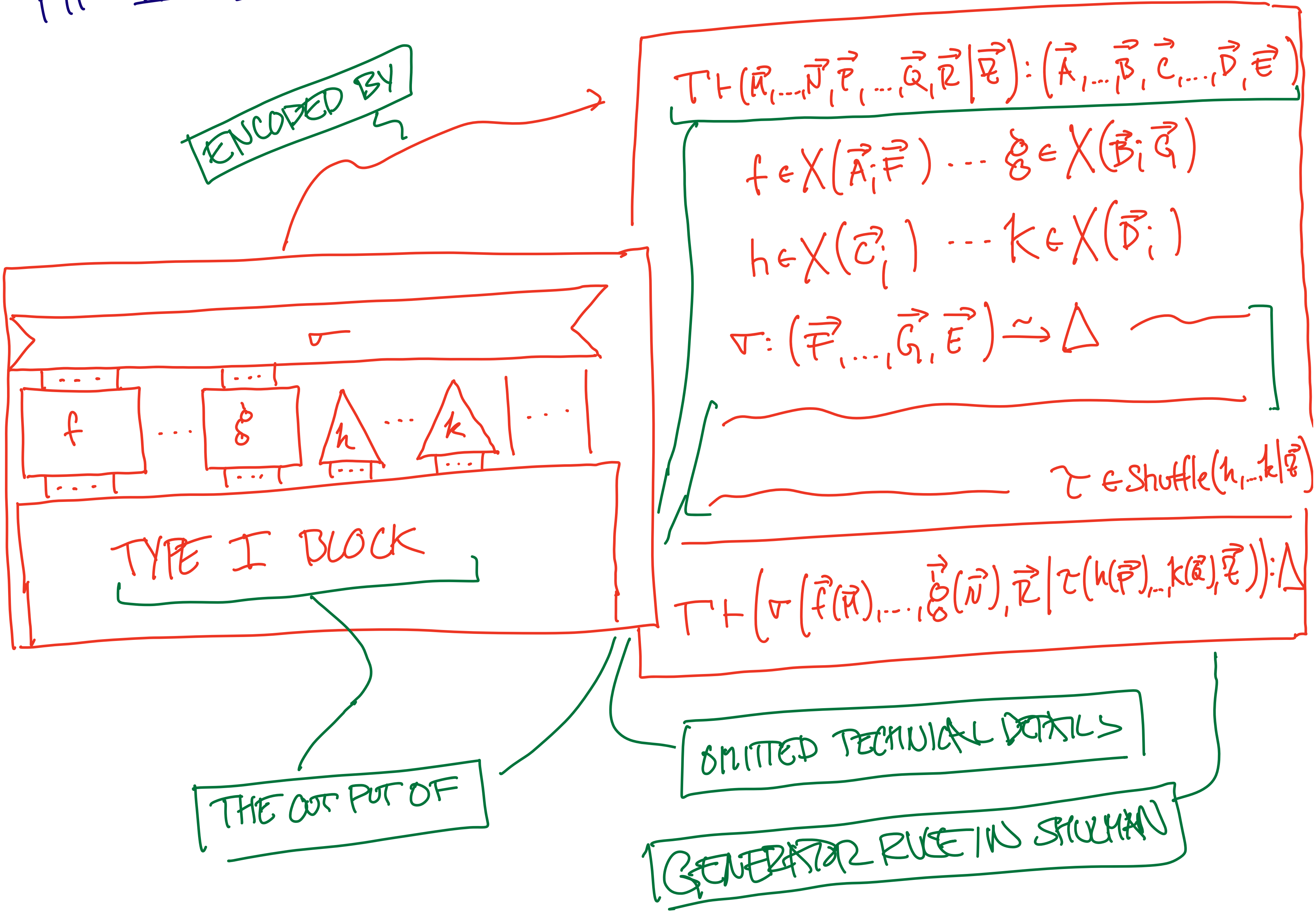


$$\frac{\begin{array}{l} f \in X(;\vec{B},1) \cdots g \in X(;\vec{C},1) \\ h \in X(;) \cdots k \in X(;) \\ a, \ldots, b \in \mathbb{Z} \quad \text{(PWISE DISTINCT)} \\ \sigma : (\vec{A}, \vec{B}, \ldots, \vec{C}) \rightsquigarrow \Delta \end{array}}{\vec{x} : \vec{A} \vdash (\sigma(\vec{x}, \vec{f}^{a}, \ldots, \vec{g}^{b}) \mid h, \ldots, k) : \Delta}$$

IDENTITY RULE IS SHOWN

TECHNICAL DETAILS OMITTED

PTT' III.III - TYPE II RULE



PTT′ III.IV – THEOREM ON SYNCAT(PTT'_X)

THEOREM (SHULMAN)

GIVEN A 1-COMPUTAD X THE CATEGORY

$SynCat(PTT'_X)$ W/ OBJECTS TYPES/CONTEXTS

MORPHISMS DER TYPING J.

$\Rightarrow$ ~ DER. EQ J.

IS EQUIV TO $P_1^T(X)$.

$\boxed{\text{NAMES OF VARIABLES}}$

$\boxed{\text{OMITTED HERE DESUGARING \& PERMUTING SCALAR TERMS}}$

PT^2 - IMPOSING EQUALITIES

OMITTED FOR TIME

•) AXIOMS FOR EACH GENERATING RELATION

••) RULES ENOUGH TO MAKE =
A CONGRUENCE

SCPTT & MPTT

SCPTT & MPTT

FOR SEMICARTESIAN CATEGORIES

FOR MARKOV CATEGORIES

ARE JUST PTT W/

TERM RULES

ONE REMOVED FOR SCPTT

ONE ADDED FOR MPTT

BECAUSE NO SCALARS AND ADDING DUPLICATION

MODIFIED TYPING RULES

SIMPLIFIED EQUALITY RULES

BECAUSE NO SCALARS

SCPTT I - TERM FORMATION RULES

TERM FORMATION RULES SO THAT

GIVES RISE TO

$$f \in X\left(\boxed{\frac{n=1}{m \geq 1}}\right) \rightsquigarrow f(\underbrace{x_1, x_2, \ldots, x_m}_{\text{PREEXISTING TERMS}}) \quad \text{TERM}$$

$$f \in X\left(\boxed{\frac{n \geq 1}{m \geq 1}}\right) \rightsquigarrow f_{(i)}(x_1, x_2, \ldots, x_m) \quad \text{TERM}$$

$\boxed{i \in \langle n \rangle}$

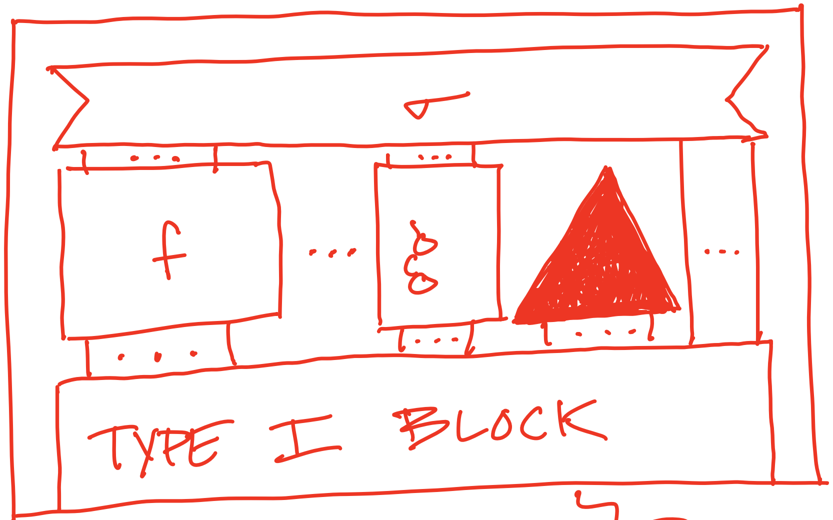
$$f \in X\left(\boxed{\frac{n=1}{m=0}}\right) \longrightarrow f^a$$

A LABEL IN AN ALPHABET (SYNTACTIC SUGAR)

$$f \in X\left(\boxed{\frac{n \geq 2}{m=0}}\right) \rightsquigarrow f^a_{(i)}$$

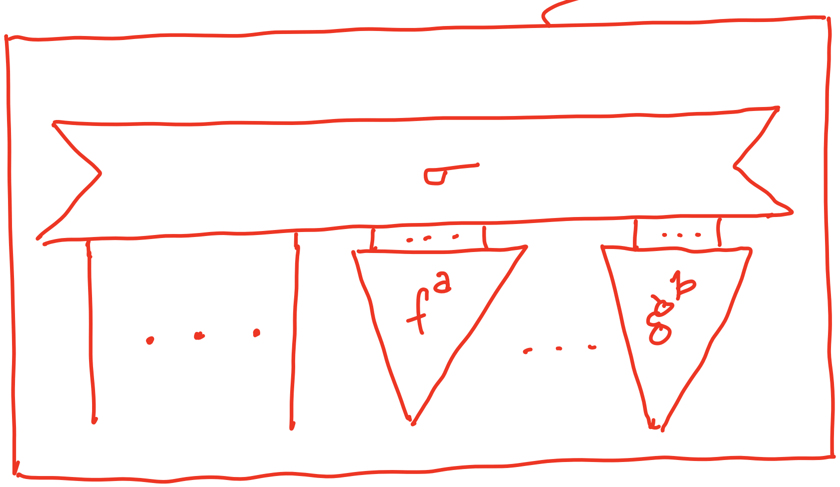
N.B. x_i THAT'S FOR US $\quad$ $x_{(i)}$ THAT'S IN THE SYNTAX

SCPTT II - TYPING RULES



$$\frac{\begin{array}{c} T \vdash (\vec{M}, \dots, \vec{N}, \vec{P}, \vec{R}) : (\vec{A}, \dots, \vec{B}, \vec{C}, \vec{E}) \\ f \in X(\vec{A}; \vec{F}) \quad \dots \quad g \in X(\vec{B}; \vec{G}) \\ \sigma : (\vec{F}, \dots, \vec{G}, \vec{E}) \xrightarrow{\sim} \Delta \end{array}}{T \vdash (\sigma(\vec{f}(\vec{M}), \dots, \vec{g}(\vec{N}), \vec{R})) : \Delta}$$

ENCODED BY



$$\frac{\begin{array}{c} f \in X(; \vec{B}_{\geq 1}) \quad \dots \quad g \in X(; \vec{C}_{\geq 1}) \\ a, \dots, b \in \mathcal{J} \quad \text{(PWISE DISTINCT)} \\ \sigma : (\vec{A}, \vec{B}, \dots, \vec{C}) \xrightarrow{\sim} \Delta \end{array}}{\vec{x} : \vec{A} \vdash (\sigma(\vec{x}, \vec{f^a}, \dots, \vec{g^b})) : \Delta}$$

MPTT - TERM FORMATION RULES

TERM FORMATION RULES SO THAT

GIVES RISE TO

$f \in X\left(\boxed{\frac{n=1}{m \geq 1}}\right) \rightsquigarrow f(x_1, x_2, \ldots, x_m)$ TERM

PREEXISTING TERMS

$f \in X\left(\boxed{\frac{n \geq 1}{m \geq 1}}\right) \rightsquigarrow f_{(i)}(x_1, x_2, \ldots, x_m)$ TERM

$\boxed{i \in \langle n \rangle}$

$c \in \mathbb{N}_{\geq 2} \rightsquigarrow X_{(i)}$ $\boxed{i \in \langle c \rangle}$

PREEXISTING TERM

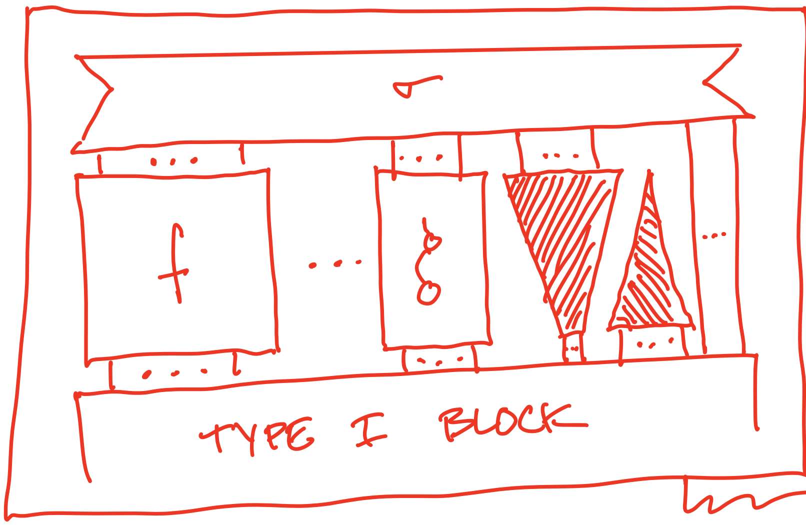
$f \in X\left(\boxed{\frac{n=1}{m=0}}\right) \rightsquigarrow f^a$

A LABEL IN AN ALPHABET (SYNTACTIC SUGAR)

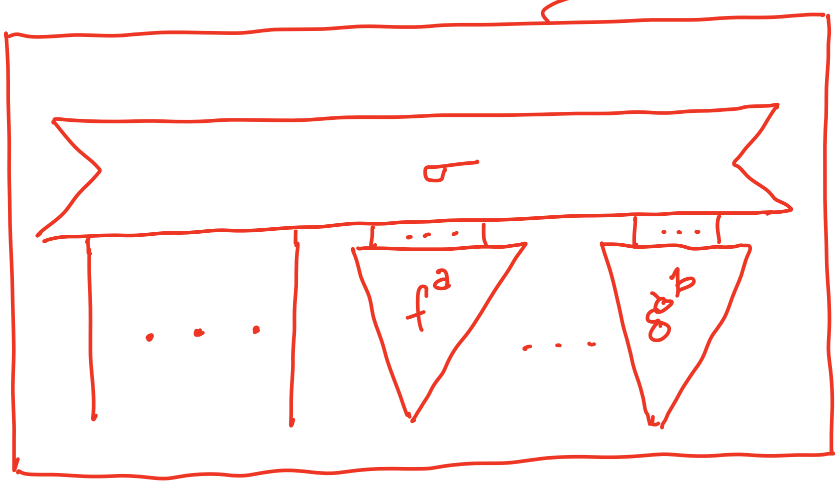
$f \in X\left(\boxed{\frac{n \geq 2}{m=0}}\right) \rightsquigarrow f^a_{(i)}$

THIS ALLOWS FOR DUPLICATION

MPTT - TYPING JUDGEMENT RULES



$$\frac{\Gamma \vdash (\vec{M}, \ldots, \vec{N}, \vec{P}, \vec{Q}, \vec{R}) : (\vec{A}, \ldots, \vec{B}, \vec{C}, \vec{D}, \vec{E}) \quad f \in X(\vec{A}; \vec{F}) \ \cdots \ g \in X(\vec{B}; \vec{G}) \quad n \in \mathbb{N} \quad \sigma : (\vec{F}, \ldots, \vec{G}, \vec{C}_{(1)}, \ldots, \vec{C}_{(n)}, \vec{E}) \rightsquigarrow \Delta}{\Gamma \vdash (\sigma(\vec{f}(\vec{M}), \ldots, \vec{g}(\vec{N}), \vec{P}_{(1)}, \ldots, \vec{P}_{(n)}, \vec{R})) : \Delta}$$



$$\frac{f \in X(; \vec{B}_{\geq 1}) \ \cdots \ g \in X(; \vec{C}_{\geq 1}) \quad a, \ldots, b \in \mathbb{N} \ \text{(PWISE DISTINCT)} \quad \sigma : (\vec{A}, \vec{B}, \ldots, \vec{C}) \rightsquigarrow \Delta}{\vec{x} : \vec{A} \vdash (\sigma(\vec{x}, \vec{f}^{a}, \ldots, \vec{g}^{b})) : \Delta}$$

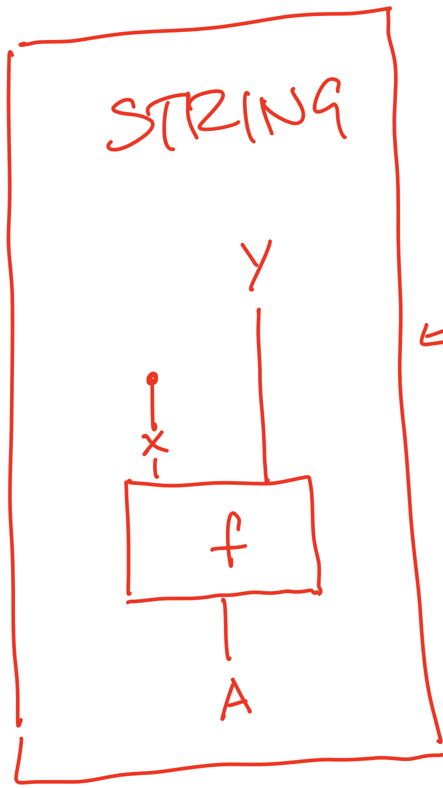
MPTT - DEFINING EQUATION OF MARGINALIZATION

$f : A \longrightarrow X \otimes Y$

IS SYNTACTIC SUGAR FOR

FRITZ

$f(y \mid a)$



MPTT

$a : A \vdash f_{(2)}(a) : Y$

CORRESPONDS TO

MPTT - CONDITIONALS I

DEFINITION

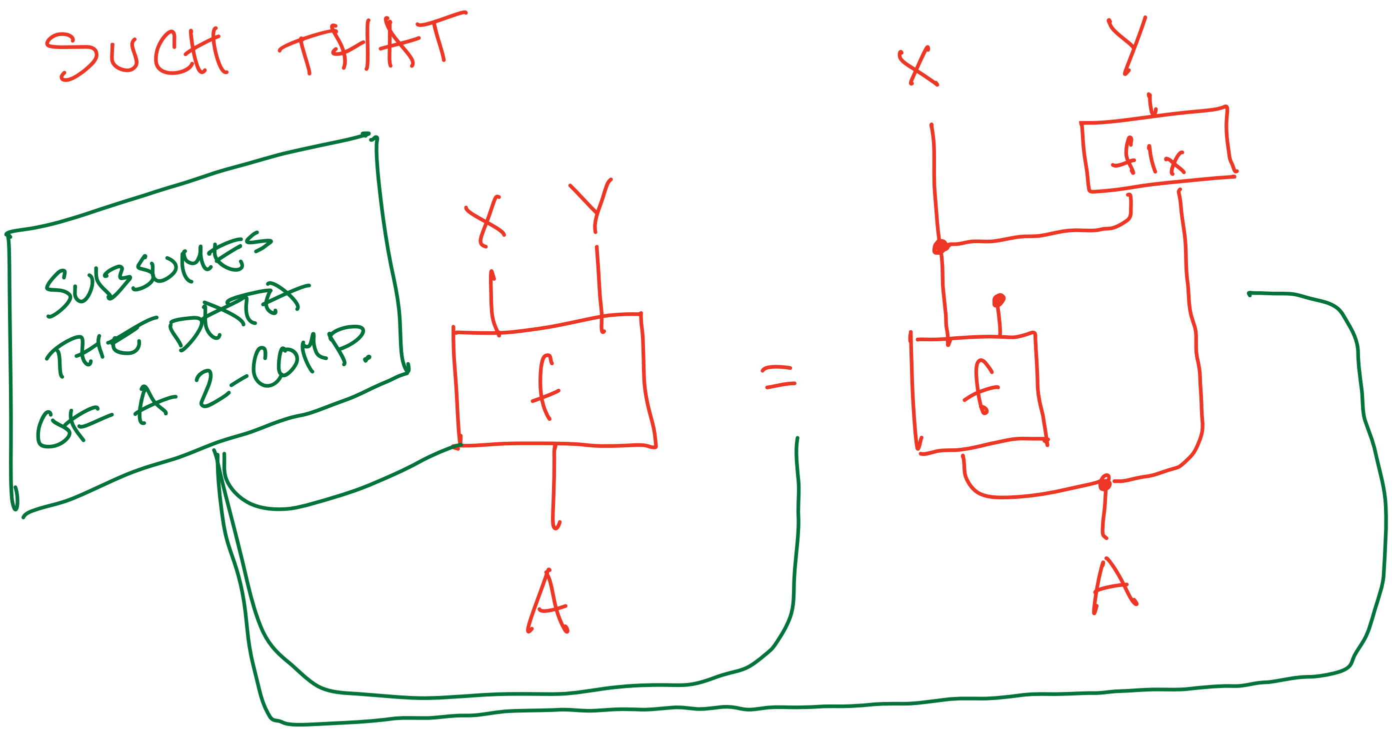
WE SAY A MARKOV CATEGORY $\mathcal{C}$ HAS CONDITIONALS IF, FOR ALL

$$f : A \longrightarrow X \otimes Y$$

THERE EXISTS

$$f|_X : X \otimes A \longrightarrow Y$$

SUCH THAT



MLTT - CONDITIONALS II

THAT EQUALITY CORRESPONDS TO THE EQ. JUDGEMENT

$$a : A \vdash \underline{\left(f_{(1)}(a),\ f_{(2)}(a)\right) = \left(f_{(1)}(a),\ f|_X\left(f_{(1)}(a)\right)\right)} : (X, Y)$$

COULD ~~BE~~ SUGARED TO MAKE MORE INTUITIVE

THE REAL BEGINNING

DESIDERATUM

A TYPE SYSTEM FOR MOLECULAR PROGRAMMING SUITABLE FOR APPLICATION TO THE LIFE SCIENCES

SYNTHESIZE

- I (VAST) NETWORKS OF CHEMICAL REACTIONS
- II CHEM. AS PROBALISTIC PROCESSES
- III SPACES OF CONFORMATIONS OF MOLECULES