Resource Sharing Machines

Frameworks for composing dynamical systems

Sophie Libkind

Setup

A. Dynamical systems model things that change

e.g.:
- me
- pendulum
- fluid
- clock
- computer
- ecosystem
- cellular automata

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- hybrid systems

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Motivation

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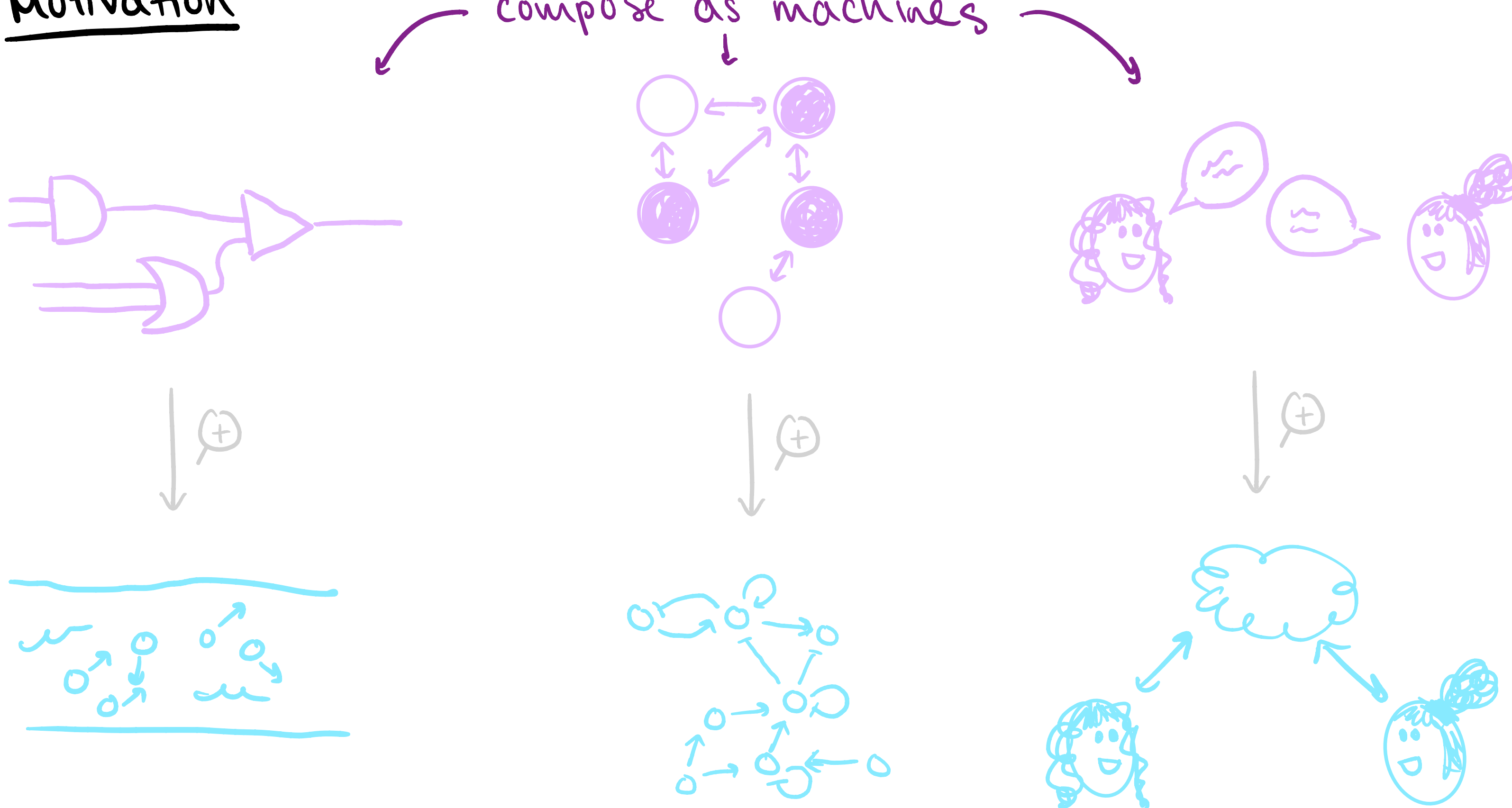
compose as machines



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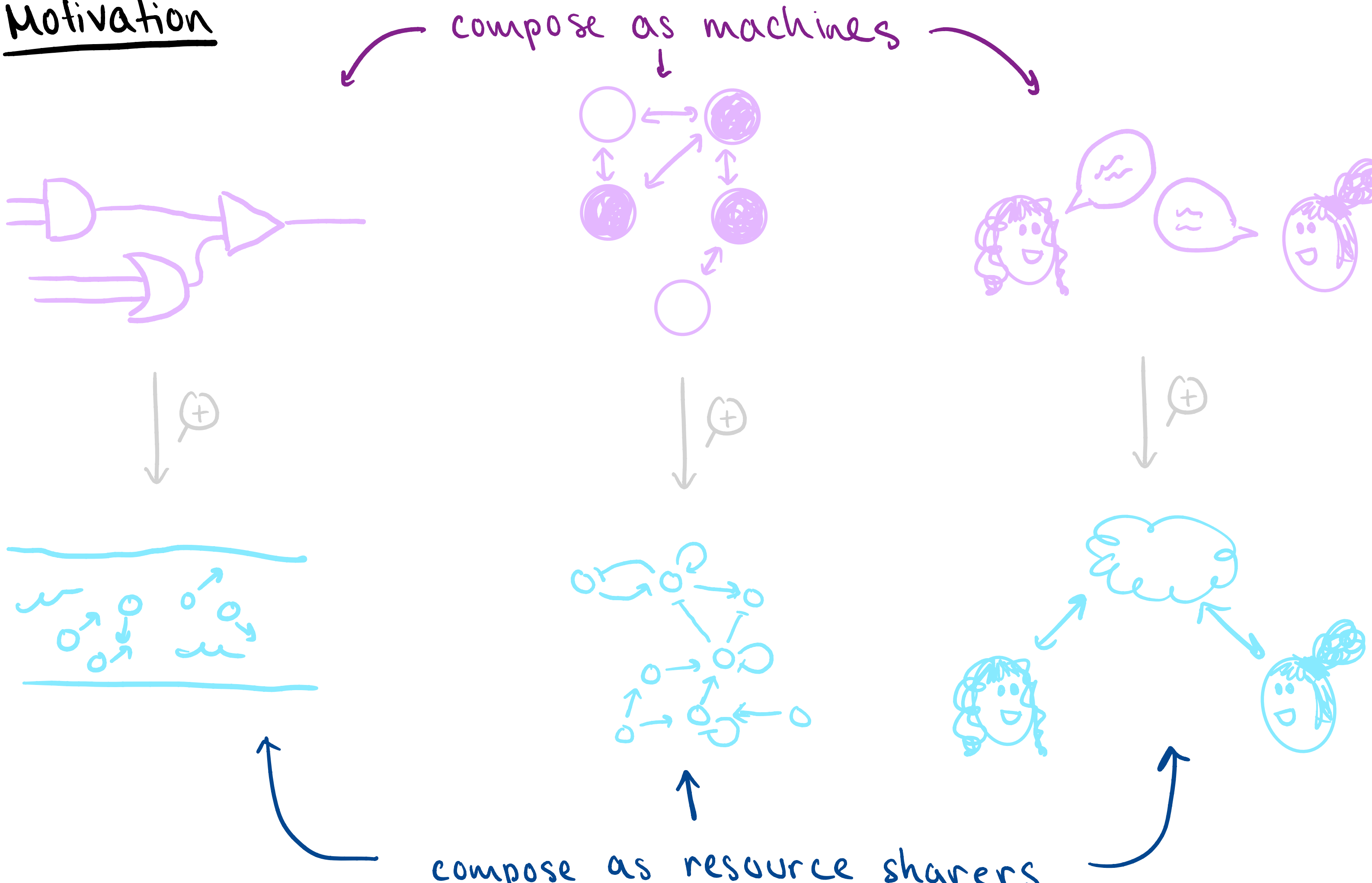

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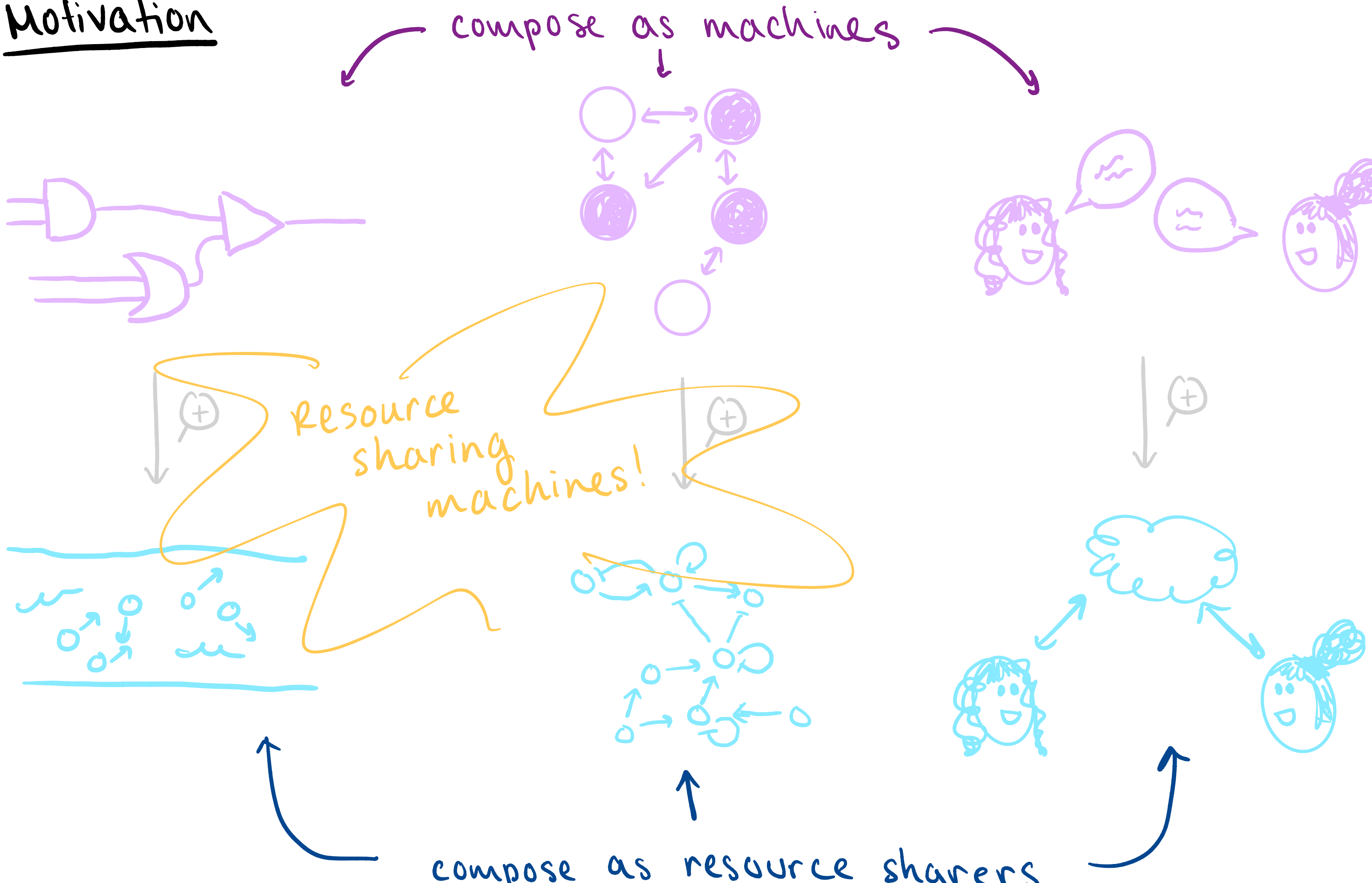
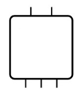
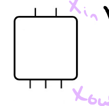

compose as machines
Resource sharing machines!
compose as resource sharers

Machines

	an operad W theory	an algebra $Dyn: W \to Set$ model
sorts		
arrange-ments		

*Let $(R, +, 0)$ a monoid

Machines

	theory (an operad $\mathcal{W}$)	model (an algebra $\mathrm{Dyn}: \mathcal{W} \to \mathrm{Set}$)
sorts	pairs - $\begin{pmatrix} X_{in} \\ X_{out} \end{pmatrix}$ e.g. $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ = 	Dyn() $= \left\{ \begin{pmatrix} S \in \mathrm{FinSet} \\ u : R^{X_{in}} \times R^S \longrightarrow R^S \\ r : R^S \longrightarrow R^{X_{out}} \end{pmatrix} \right\}$
arrange-ments		

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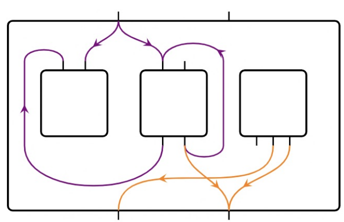
Machines

	theory an operad $\mathcal{W}$	model an algebra $\mathrm{Dyn}: \mathcal{W} \to \mathbf{Set}$
sorts	pairs - $\binom{X_{in}}{X_{out}}$ e.g. $\binom{2}{3}$ = [box]	$\mathrm{Dyn}\left(\text{[box]}\ {}^{X_{in}}_{X_{out}} \right) = \left\{ \left(\begin{array}{l} S \in \mathrm{FinSet} \\ u: R^{X_{in}} \times R^S \to R^S \\ r: R^S \to R^{X_{out}} \end{array} \right) \right\}$
arrange-ments		pass information along wires

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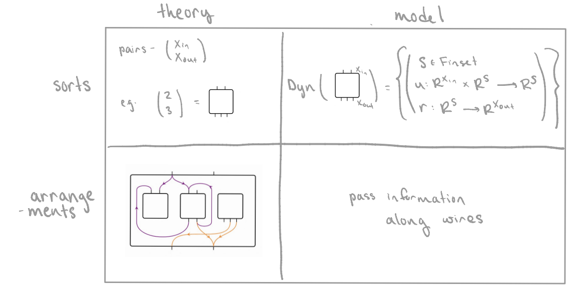
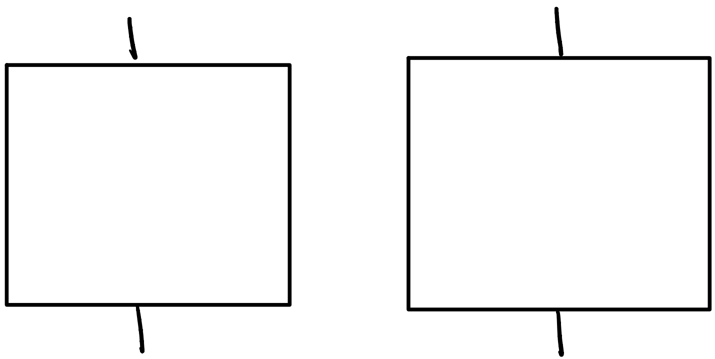
Machines - example

$(\mathbb{R}, +, 0) = (\text{Bool}, \text{or}, \text{false})$





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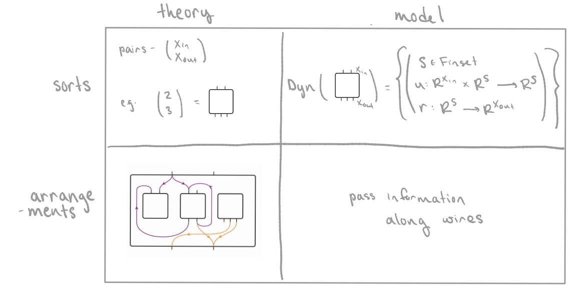
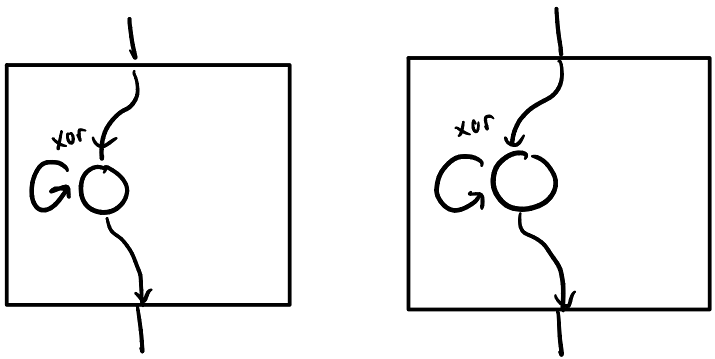
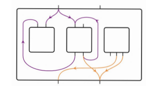


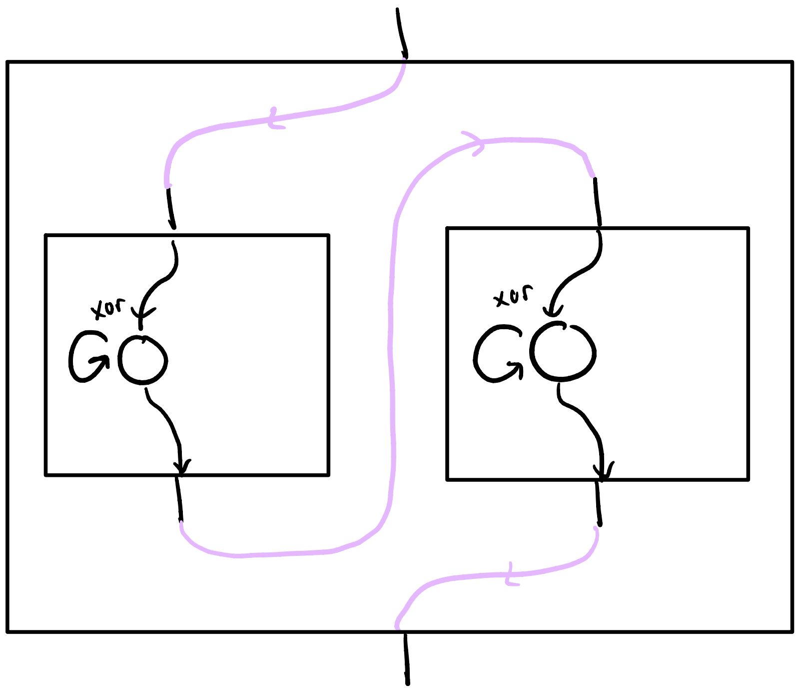


$$u: \text{Bool} \times \text{Bool} \longrightarrow \text{Bool}$$
$$(a, s) \longmapsto a \text{ xor } s$$

Machines - example

$(R, +, 0) = (\mathrm{Bool}, \mathrm{or}, \mathrm{false})$

	theory	model
sorts	pairs - $\begin{pmatrix} X_{in} \\ X_{out} \end{pmatrix}$ e.g. $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ = [box]	Dyn([box]) = $\left\{ \begin{pmatrix} S \in \mathrm{Finset} \\ u : R^{X_{in}} \times R^S \to R^S \\ r : R^S \to R^{X_{out}} \end{pmatrix} \right\}$
arrange-ments		pass information along wires



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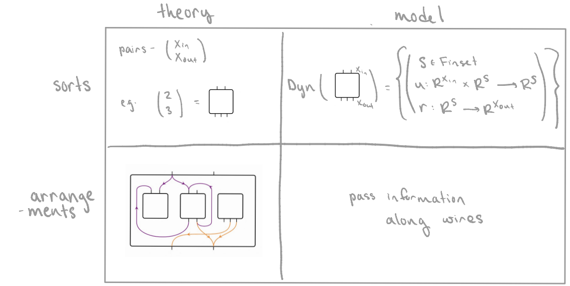

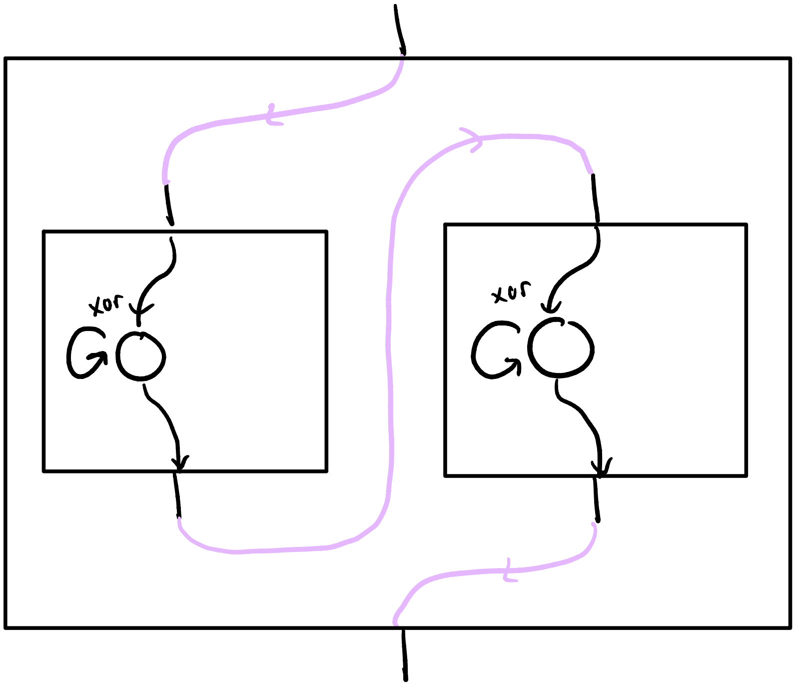

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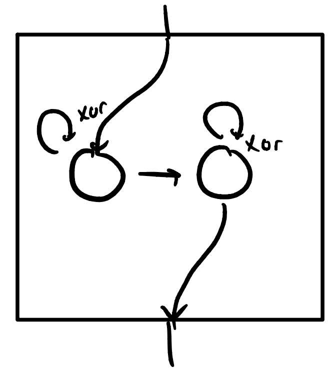

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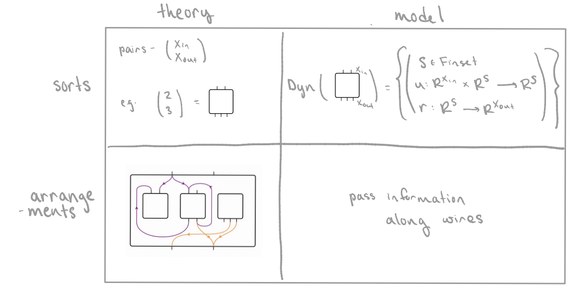

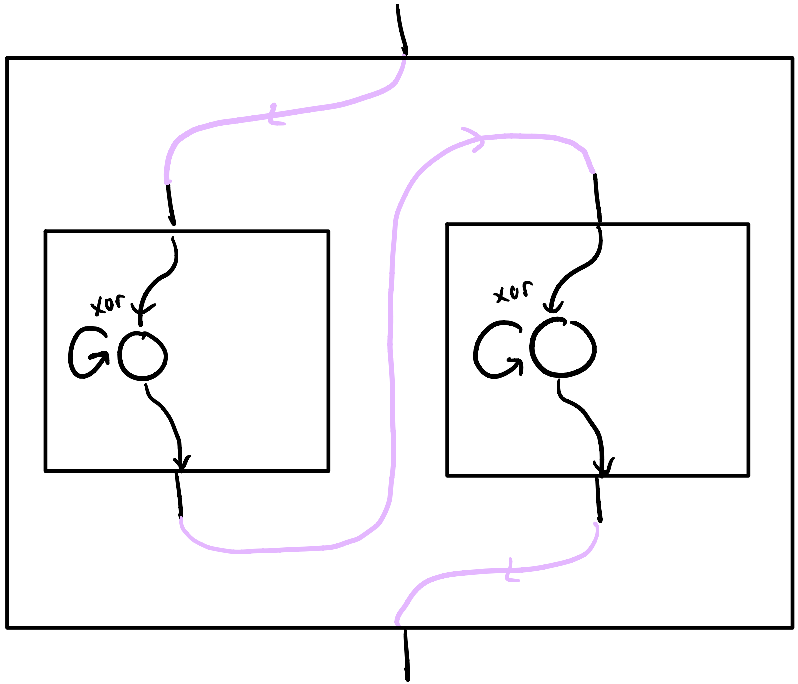

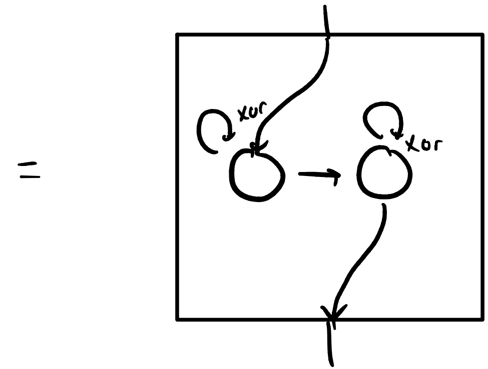

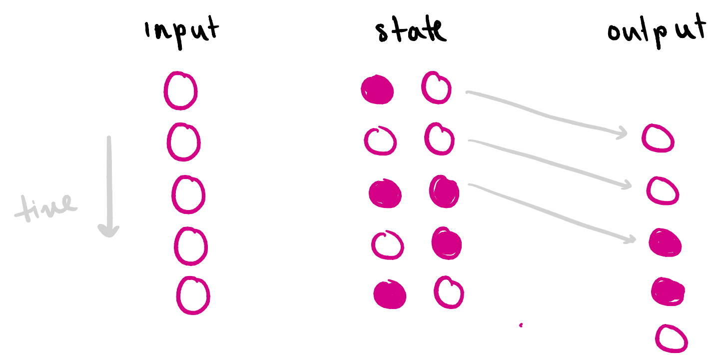

Resource Sharers

an operad $\mathcal{O}(\mathrm{Cspn})$ an algebra $\mathrm{Dyn}: \mathcal{O}(\mathrm{Cspn}) \to \mathrm{Set}$

	theory	model
sorts		
arrange-ments		

*Let $(R, +, 0)$ a monoid

Resource Sharers

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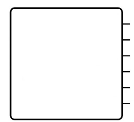
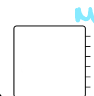
sorts	Finite sets - M $G =$ 	$\mathrm{Dyn}\Big($  $\Big) = \left\{ \begin{pmatrix} S \in \mathrm{Finset} \\ u: R^S \longrightarrow R^S \\ p: M \to S \end{pmatrix} \right\}$
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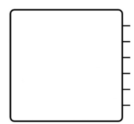
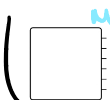
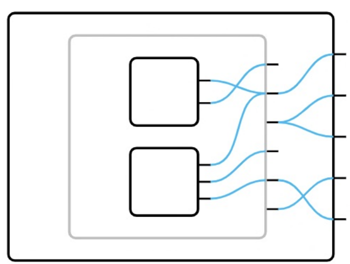
sorts	Finite sets - M G = 	Dyn() = $\left\{ \begin{pmatrix} S \in \mathrm{Finset} \\ u: R^S \to R^S \\ p: M \to S \end{pmatrix} \right\}$
arrange-ments		identify shared state vars and add the effects

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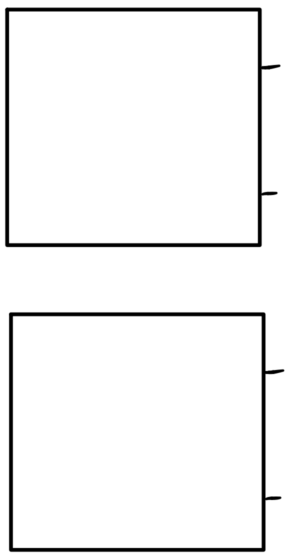
Resource sharers - example



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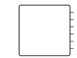
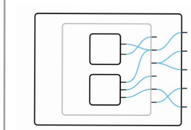
sorts	Finite sets - M $G =$ 	$\mathrm{Dyn}(\square^M) = \left\{ \begin{pmatrix} S \in \mathrm{Finset} \\ u : \mathbb{R}^S \longrightarrow \mathbb{R}^S \\ p : M \longrightarrow S \end{pmatrix} \right\}$
arrange-ments		identify shared state vars and add the effects

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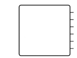
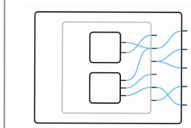
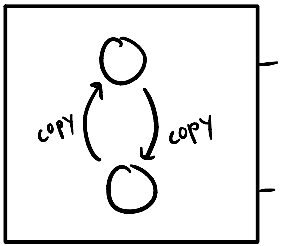
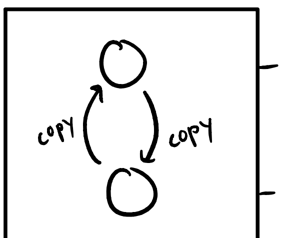
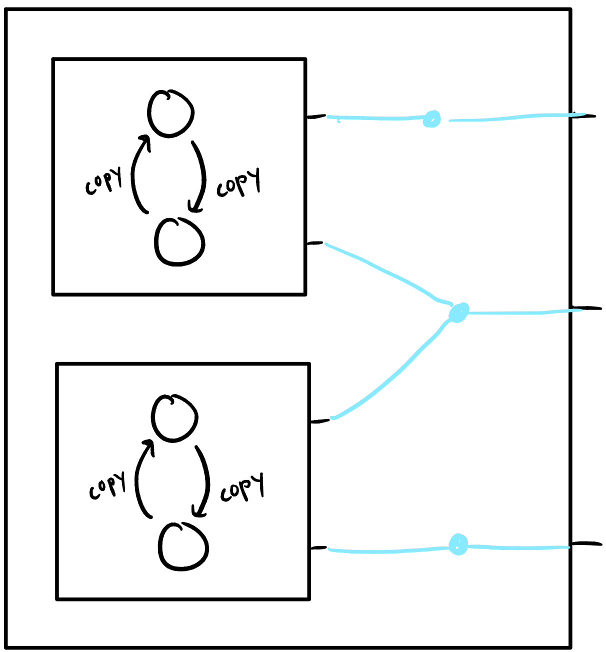
sorts	Finite sets - M $G =$ 	$\mathrm{Dyn}(\square^{M}) = \left\{ \begin{pmatrix} S \in \mathrm{Finset} \\ u: \mathbb{R}^S \longrightarrow \mathbb{R}^S \\ p: M \longrightarrow S \end{pmatrix} \right\}$
arrange-ments		identify shared state vars and add the effects





Resource sharers-example



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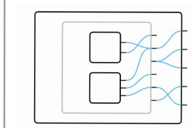
arrange-ments		identify shared state vars and add the effects

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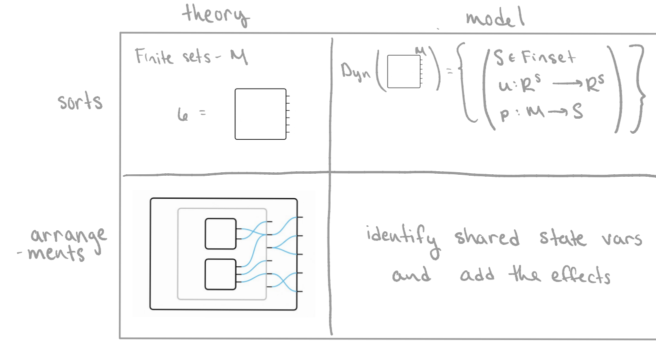
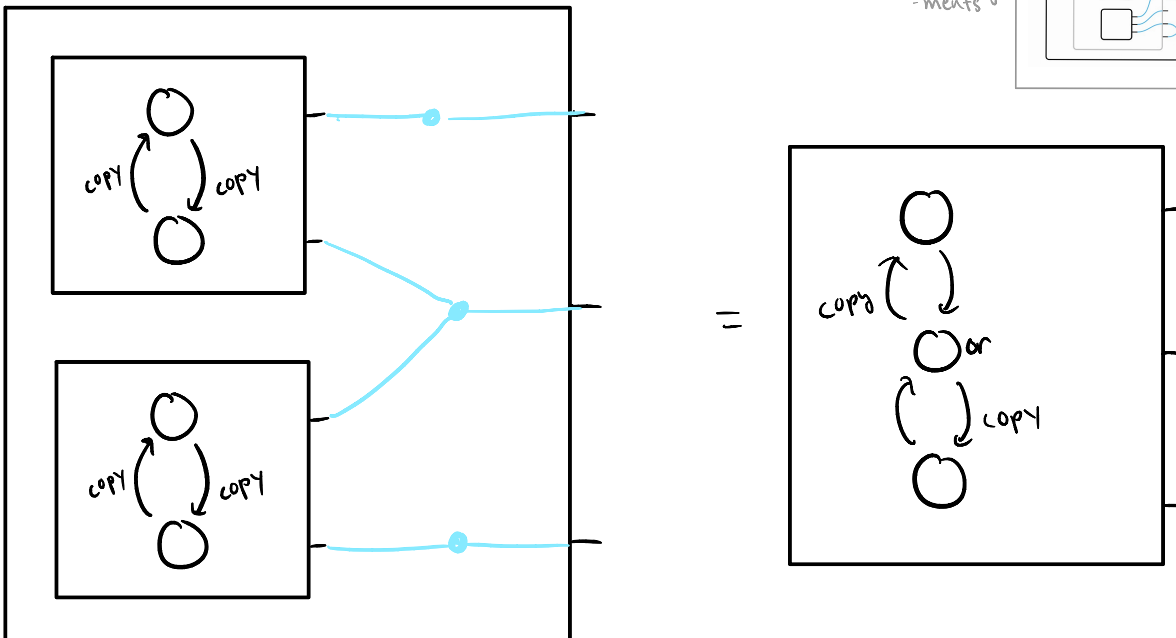
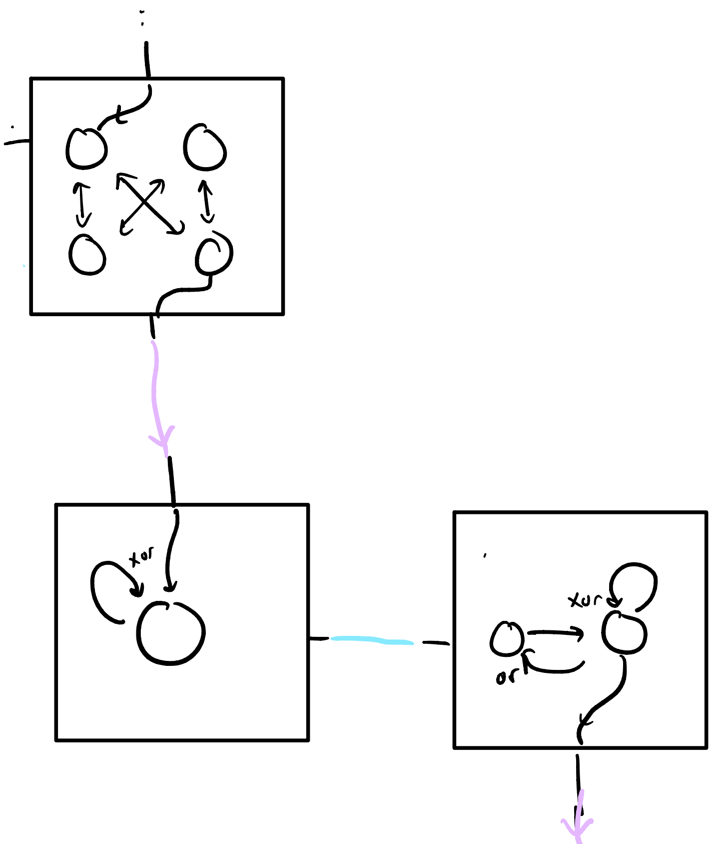




Resource sharing Machines!



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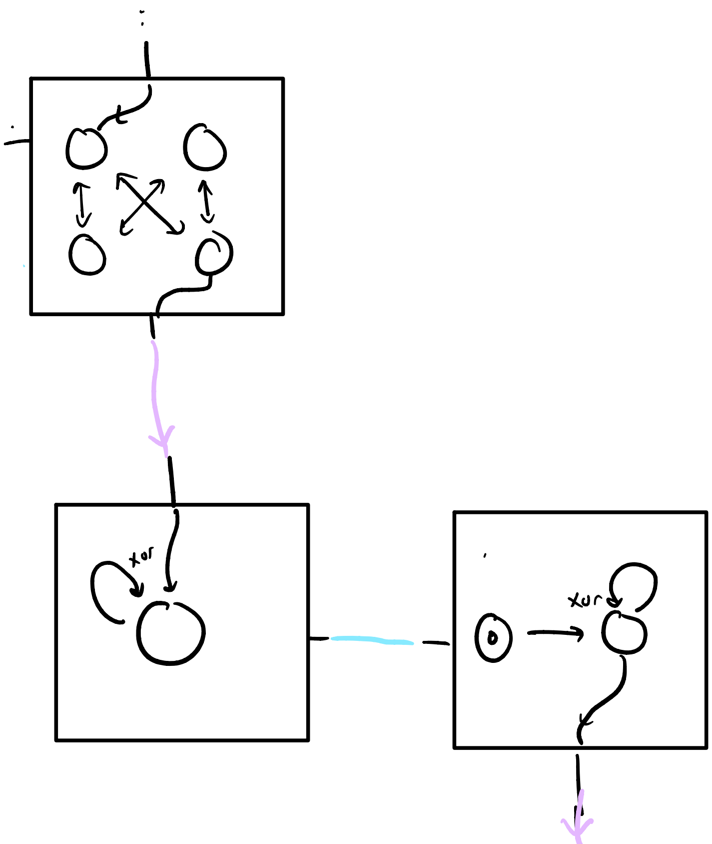

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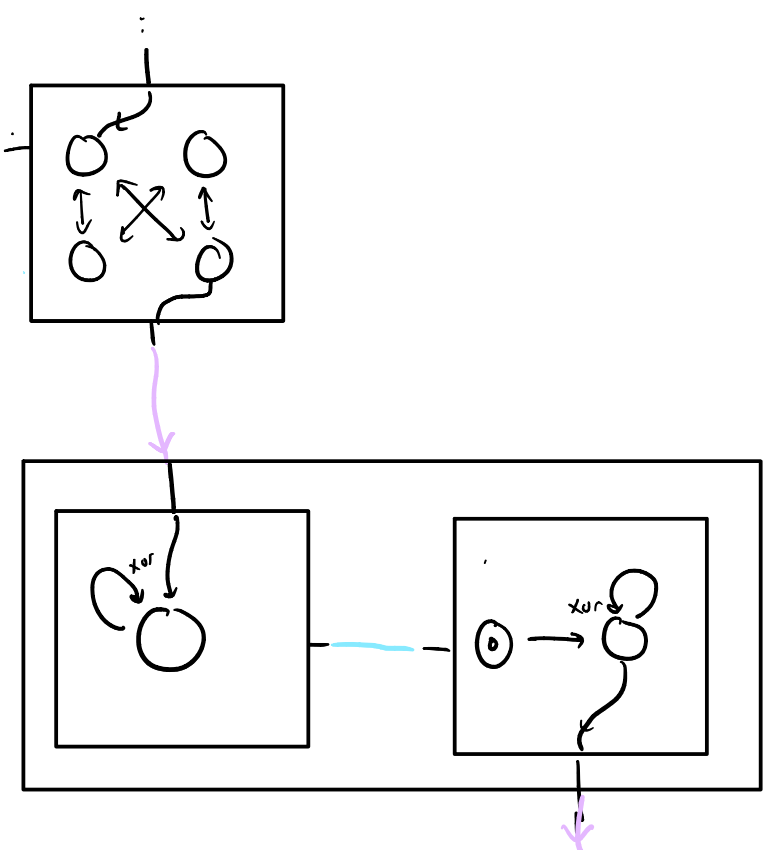

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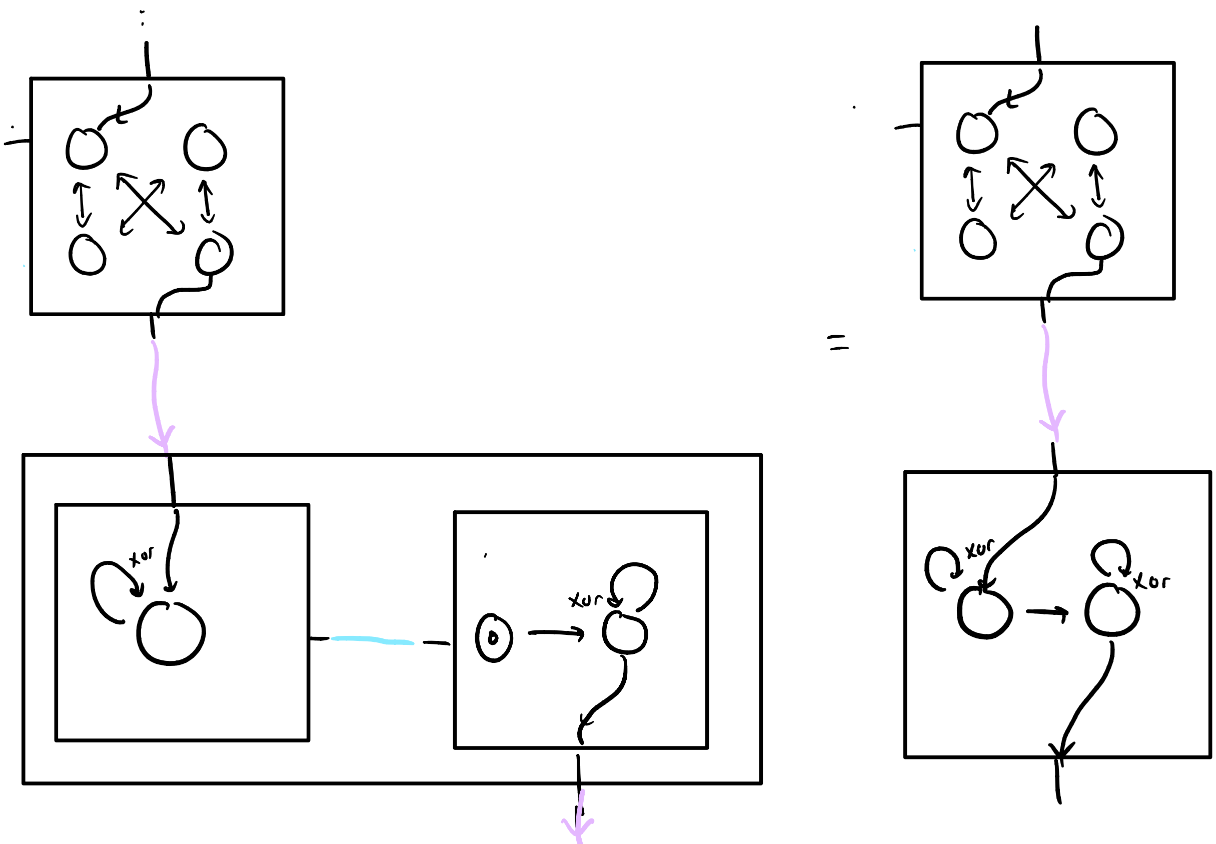

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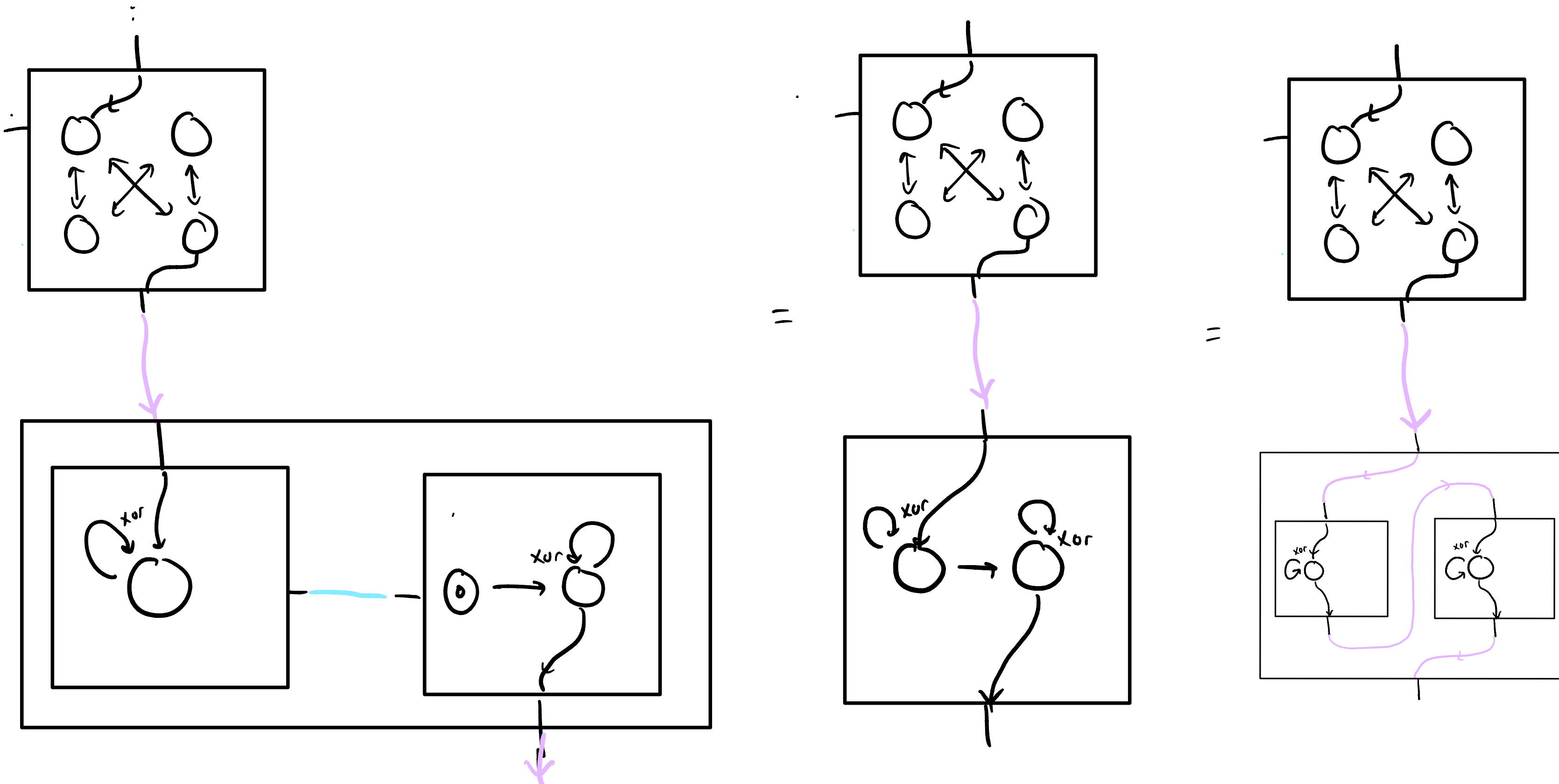

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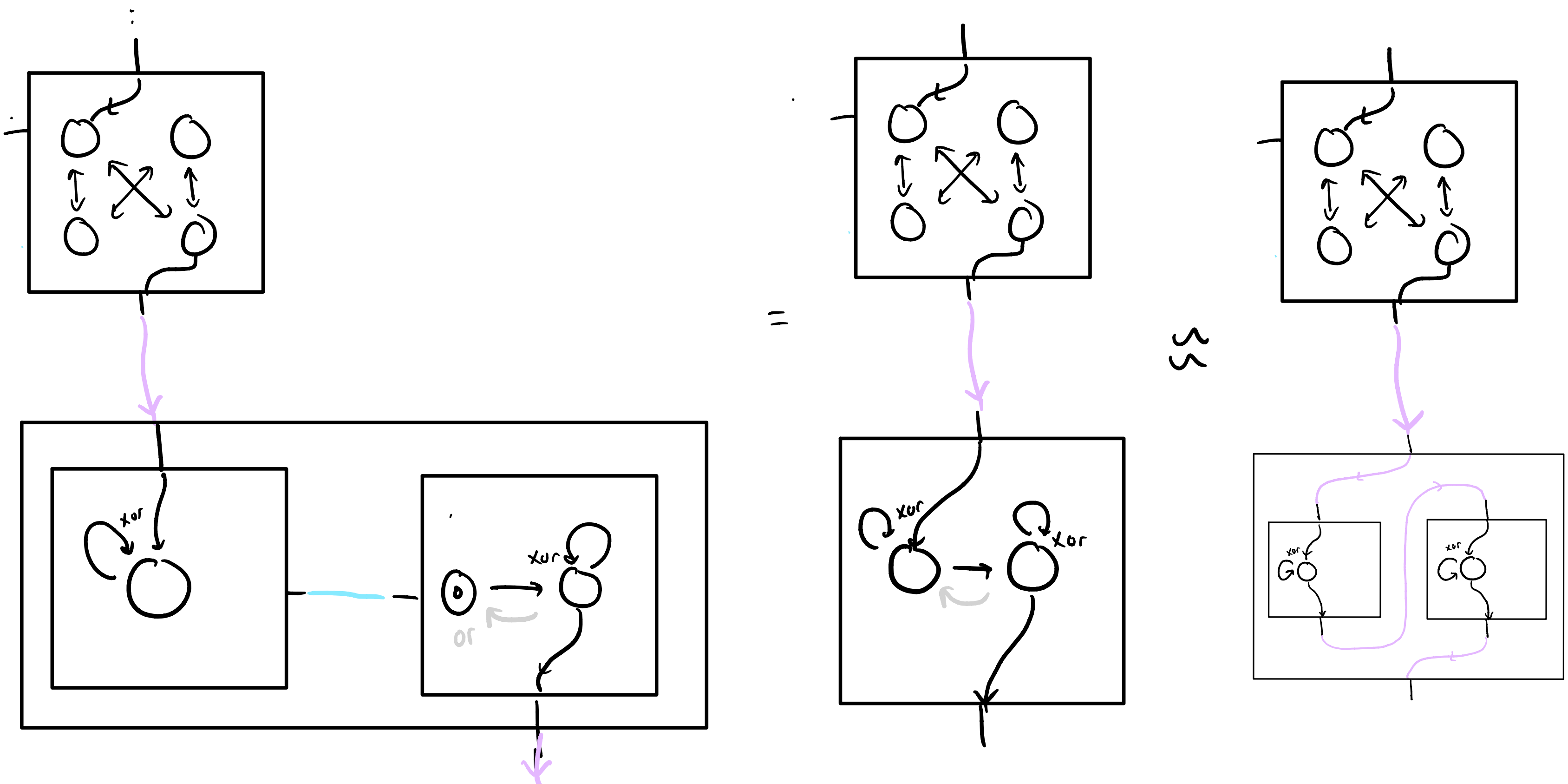
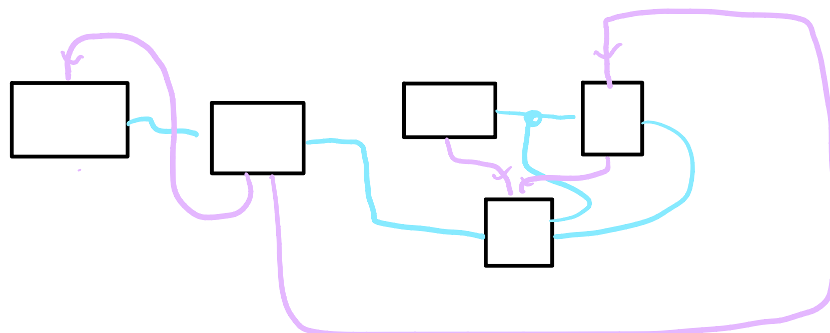


Questions | Conjectures

- A "thing" does mostly machine-y connections with other "things"



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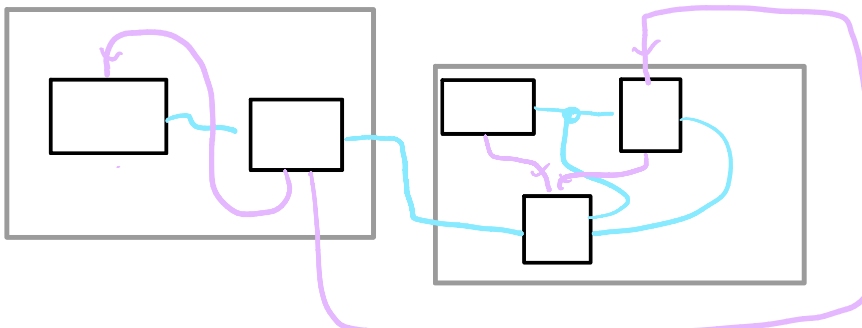

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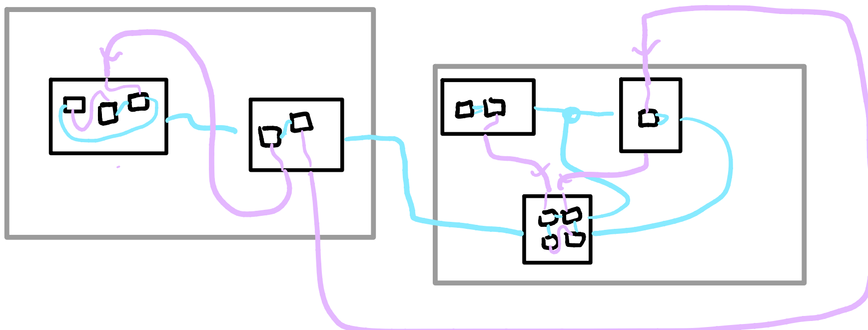

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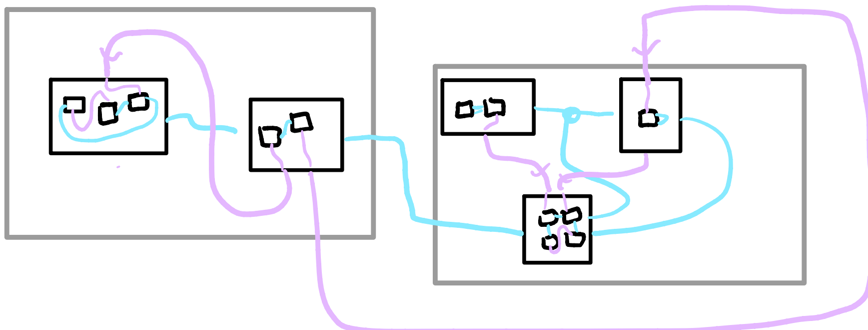

- How to mathmetize "mostly machine-y"? via between-box statistics?

- What does this have to do with information at a distance?

Questions | Conjectures

- We have different types of compositions in the same story

  ... what about different types of dynamics

Review - operads + their algebras

	an operad $\mathcal{O}$ theory	an algebra $A : \mathcal{O} \to \mathsf{Set}$ model
sorts	$\underset{NP}{\_\_\_}$ $\underset{adj}{\_\_\_}$ $\underset{VP}{\_\_\_}$ $\underset{sentence}{\_\_\_}$ ...	$A(\underset{NP}{\_\_\_}) = \{$ english noun phrases $\}$
arrange-ments	$\underset{NP}{\_\_\_}$ $\underset{VP}{\_\_\_}$ the $\underset{adj}{\_\_\_}$ $\underset{NP}{\_\_\_}$ likes to $\underset{VP}{\_\_\_}$	$A($the $\underset{adj}{\_\_\_}$ $\underset{NP}{\_\_\_}$ likes to $\underset{VP}{\_\_\_})$: $A(\underset{adj}{\_\_\_}) \times A(\underset{NP}{\_\_\_}) \times A(\underset{VP}{\_\_\_}) \to A(\underset{sent}{\_\_\_})$ "fill-in the blanks"

Nesting