$\left\{ \begin{array}{l} \text{Container} \\ \text{Polynomial functor} \end{array} \right\}$ & inductive types

1. Initial algebras / terminal coalgebras

- monads for effects

$T : \underline{\mathrm{Set}} \to \underline{\mathrm{Set}}$

$$\begin{tikzcd} T(A) \arrow[r, "f"] & A \\ T(\mu T) \arrow[u, dashed, "T(\mathrm{fold}\, f)"] \arrow[r, "in_T"'] & \mu T \arrow[u, dashed, "\mathrm{fold}\, f"'] \end{tikzcd}$$

fold f ← catamorphism

generic patterns : fusion theorems

2. Containers

Abel, A. 199 A λ-calculus with ind/coind types

$\mu X . \mathbb{N} \times X + \mathbb{N} \to X$

$\cancel{\mu X . (X \to 2) \to 2}$

strict positivity

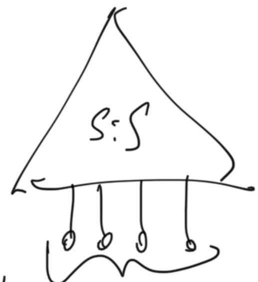
Strong Normalisation

set-based (Aczel)

Ghani, Abbott

$S : Set \;,\; P : S \to Set$

$(S \lhd P) : Set \to Set$



$(S \lhd P)\, X = \Sigma s : S . P s \to X$ P(s)

$S = \mathbb{N} \quad P n = \{0, 1, .., n-1\}$

List

$S =$  $P n$ counts leaves

counts nodes

$\int_{X : Set} (S \lhd P)\, X \to (T \lhd Q)\, X$ large

$\cong \Sigma f : S \to T, \Pi s : S . Q(f s) \to P s$

small

A.

Cont is CCC (Staton, Levy)

$$(S \triangleleft P) \circ (T \triangleleft Q)$$

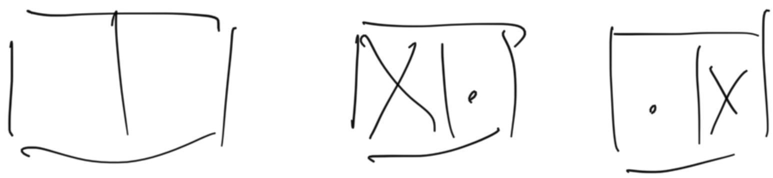
$$= \sum s : S;\ f : P_s \to T \ \triangleleft \ \sum p : P_s,\ Q(f\,p)$$

3. Derivatives

Zipper

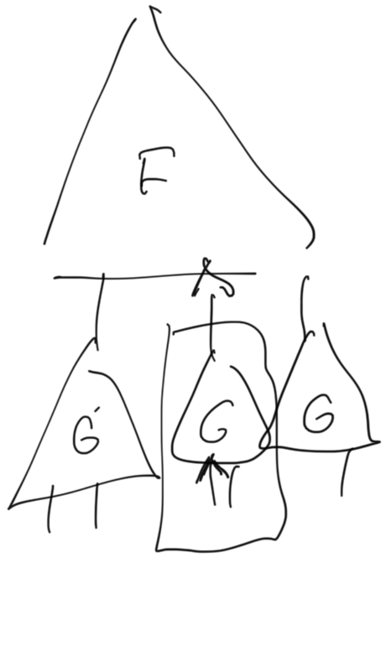
Type of 1-hole contexts

$$F(X) = X \times X = X^2 \qquad \partial F(X) = X + X = 2 \times X$$



$$\partial(F \circ G)\, X$$

$$= \partial F(G(X)) \times \partial G(X)$$



$$\partial(S \triangleleft P)\, X =$$

$$\sum s:S, p:P(s) \lhd \sum q:P(s).\ p \neq q$$

Δ

Typification of used formulas

dev. of polynomials

∂ for data

$$\cancel{= \underbrace{\text{Cart}(\mathbb{I}, S \lhd P)}_{\text{Cart.}}} \ \cancel{\ } \quad \text{container} \quad ?$$

H. Gylterud „Symmetric container"

What is the antiderivative?

$\text{Cart}(F, \partial G)$

$= \text{Cart}(F \times \mathbb{I}, G)$ $\checkmark$ or $\partial G = \mathbb{I} \multimap G$

$$\partial(S \lhd P) = \text{List}\ ?$$

$$\partial\, \text{List} = \text{List} \times \text{List}$$

Groupoid container

$S : \text{Groupoid}$

$P : S \to \underline{\text{Set}}$

$(S \lhd P) : \underline{\text{Set}} \to \underline{\text{Set}}$

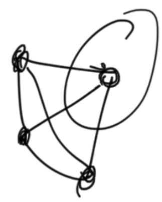
$$(S \lhd P)X = \int^{s:S} P_s \to X$$

$MS = S : \mathrm{Obj}\ \mathbb{N}$

$\cancel{S}S(m,n) = \mathrm{Fin}\ m \simeq \mathrm{Fin}\ n$

$\lhd\ P(n) = \mathrm{Fin}\ n$

$\partial\, MS = MS$

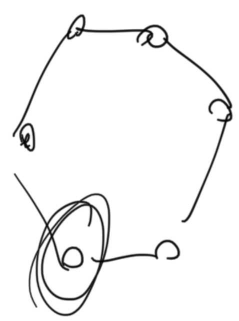


Cycles $|S| = \mathbb{N}$

$S(m,n) = i \leq n$ cyclic groups

$P(n) = \mathrm{Fin}\,(n)$

$P(i) = j \mapsto i + j \bmod n$



∂ Cycles $=$ List

C^n has antider. if $n+1$ is the size finite field $\Big\}$ C. Sattler

C^5

$(S \lhd P) \circ (T \lhd Q) = ?\ \boxed{AC}$

HoTT $\quad S : 1\text{-Type}$

$$\frac{P : S \to Set}{S \triangleleft P : Set \to Set}$$

$$(S \triangleleft P)\, X = \| \Sigma s : S.\ P s \to X \|_0$$

- Combinatory species

4. HIIT (HoTT)

Cauchy Reals = Cauchy completion of $\mathbb{Q}$

$\mathbb{R}$: Set

$=$

$\leq$

Partiality monad $A_\perp$: Set

QIIT

A : Set

$B : A \to Set$

Intrinsic syntax of TT

$Con : Set$

$Ty : Con \to Set$

$Tm : (\Gamma : Con) \to Ty\ \Gamma$

$Subst : Con \to Con \to Set$

$\circ\ ,\ id$

$idr : f \circ id = f$

$\_,\_ : \boxed{(\Gamma : Con) \to Ty\ \Gamma} \to \boxed{Con}$

$\Pi : \boxed{(\Gamma : Con)(A : Ty\ \Gamma)(B : Ty\ (\Gamma , A))} \to \boxed{Ty\ \Gamma}$

Dijkstra, Kraus, Capriotti

$$\begin{array}{l} A_0 \xrightarrow{c_0} A_1 \xrightarrow{c_1} A_2 \to \dots \\ \parallel \\ Fam \end{array}$$

$A : Set \quad = Set^{\to}$

$B : A \to Set$

$c_i : A_i \to A_{i+1}$

$\cancel{x} : A_i$

$c_i : (x : L X) \to R(X, x)$

$L : A_i \to \mathrm{Set} \qquad L = S \triangleleft P$

$R : \int L \to \mathrm{Set} \qquad R = S \triangleleft Q$

+ restr. on R, C

$S \triangleleft P : C \to \mathrm{Set}$

$S : \mathrm{Set}$

$P : S \to C$

$(S \triangleleft P) : C \to \mathrm{Set}$

$(S \triangleleft P)\, X = \Sigma s : S.\, C(P s, X)$

$(1 \to \mathrm{Set}) \to (7 \to \mathrm{Set})$

$((1 \to \mathrm{Set}) \times 7) \to \mathrm{Set}$

Syntax (Refinement of Kaposi & Kovacs)
for QIIT

$$\begin{tikzcd}
c \arrow[r] \arrow[d, dash] & c, c_0 \arrow[r] \arrow[d, dash] \arrow[r, bend right, "F"'] & c, c_0, c_1 \arrow[r] \arrow[d, dash] & {} \\
TT & \dot{T}T & \mathring{T}T &
\end{tikzcd}$$

Conjecture: Complete for semantics

Universal QIIT.

? Is there simple QIIT   QW