An Idea of Creating (compositional) Particle Filtering Dynamical Models in Poly

Xiaoyan Li

1 Introduction

Machine learning algorithms have been widely incorporated in dynamic simulation modeling to incorporate with observed data. In this report, I plan to incorporate the applied category theory, especially polynomial functors to the models incorporating machine learning algorithm PF (Particle Filtering) in dynamic simulation modeling. Figure 1 shows a simple introduction of a particle filtering model in SEIR models.

2 Idea : Applying selection category in Particle Filtering Dynamic simulation modeling

This work is inspired by the *selection category* proposed by Spivak[Dav]. Based on Spivak[Dav]'s theory, we need two things to construct a selection category (Section 2 in [Dav]). And those two things are:

1. A category C carries all the information;
2. A polynomial p provides the interface for selecting from category C.

Then, the design of the selection category in Particle Filtering algorithm is listed as follows:

1. The category C carries all the possible states of a PF state space model, and possible transitions among those model states:
   (a) Objects: are all possible states of a PF state space model. For example, $[S, I, R, ln(c\beta)]$ is an object in C for an SIR (Susceptible-Infected-Recovered) PF model.
   (b) Morphisms: are SDEs (Stochastic Differential Equations) updation after n steps calculation between two states (objects). id_c is a map from a state to itself.

   So, the definition of C is Sy^S, where S is the set of all the possible state spaces. Those state spaces can also be think of as all the states of each particle as a time.

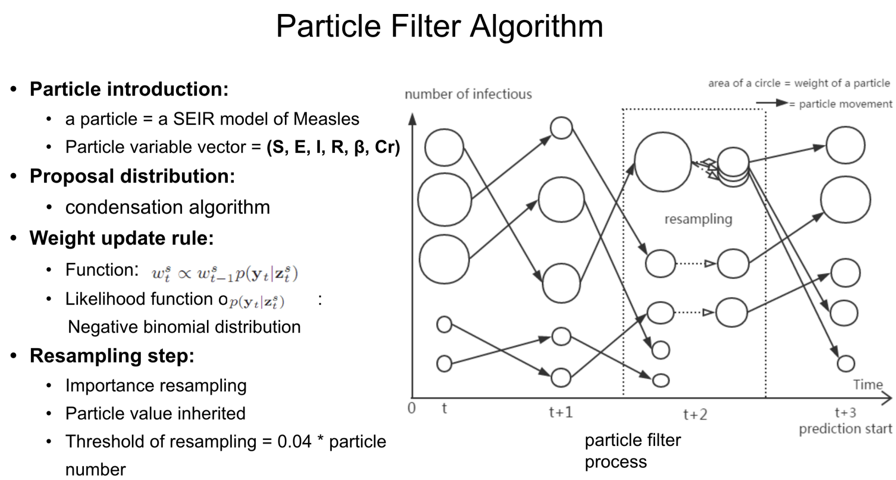

Figure 1: A simple introduction the PF algorithm in an SEIR model. And readers interested in the detailed introduction of this model please refer to my previous publication [LDO18].

2. **The polynomial p can be defined as $\Delta_N y^N$.** It indicates that the selective category will select N objects from the category C (which is a particle, also a model state, at a time t). And the particle's weight is carried by the position of p, which is Δ_N.

Finally, the selective category for the PF is $\begin{bmatrix} p \\ p \triangleleft C \end{bmatrix}$.

Then, finally, the PF machine can be defined as a dynamical system:

$$\begin{bmatrix} \Delta_N y^N \\ p \triangleleft S y^S \end{bmatrix} \otimes \left[\Delta_N y, N^N y\right] \to y^{Observation} \tag{1}$$

Where[1] $\left[\Delta_N y, N^N y\right]$ indicates a map $\Delta_N \to N^N$.

Finally, figure 2 shows design of incorporating the particle filtering algorithm designed based on this idea to the (compositional) dynamical system framework in Poly. The whole system is introduced as follows:

1. **Step 1**: We can create complex dynamical systems by composing small pieces. The example shown in figure 2 is based on the idea of composing dynamical systems as machines (details please refer to Sophie Libkind's work [Lib20]). Our previous work of composing dynamical systems with Stock and Flow diagram framework can also work in this state (details refers to the work [BLL+22]).

[1] This idea of the Poly system design is got from the idea with the discussion of David and Eric during the workshop. I did not get chance to pin down the detailed math. There should need modification and updating in the future

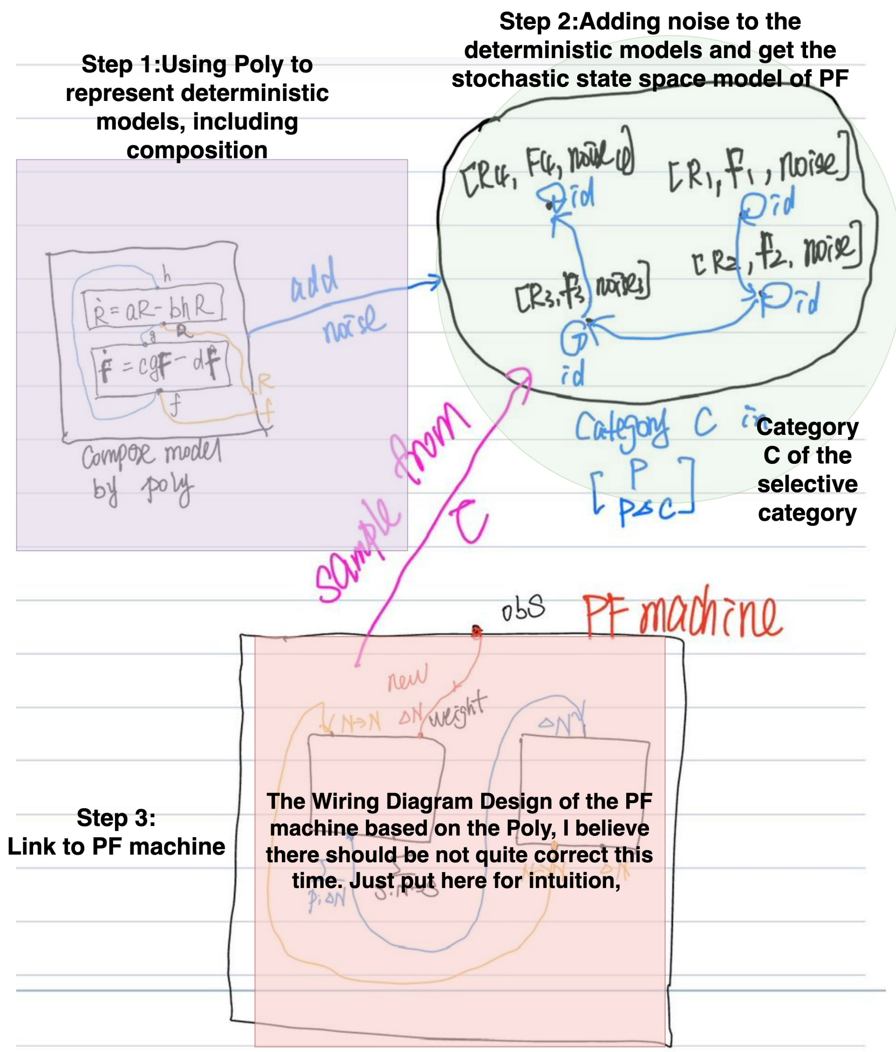

Figure 2: The idea of (composional) PF model designed in Poly.

2. **Step 2**: We can generate the state space model of PF model algorithm by adding noise to the deterministic models generated from step 1. And all the possible states of the stochastic state space model can generate the category C used in the *selective category*.

3. **Step 3**: By the idea of showing in Equation 1, we can create a PF machine. This machine samples particles from Category C created in step 2, and can update forward by observed data goes in.

3 Acknowledgement:

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References

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