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Outline

1 Introduction

- Why am I here?
- Accounting systems
- Toward PCE3
- Plan for the talk

2 Dynamical systems and interaction patterns

3 Dynamic organizational structures

4 Conclusion

Why am I here?

The question of what has brought us to this moment is always relevant.

- There are many lenses for the question "why am I here?"
- Physically, why have our atoms assembled here?
- Culturally, why are we allowed (invited / funded) to be here?
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- My understanding of PCE3 is very limited; mainly Eric Smith videos:
- Proto-life was the minimally complex "lab" that could perform...
- ...certain exothermic reactions (e.g. $CO_2 + 2H_2O \rightarrow CH_4 + 2O_2$).
- Life develops and maintains equipment for dissipating free energy.
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 - Our "funding" works like a siphon for dissipating free energy.
- I'm fascinated by nature's capacity to develop understanders like us.
 - Our prebiotic origin gives clues as to how this development occurs.
 - I would like to make such ideas formal and systematic.

I think of mathematical fields as accounting systems.

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 - We can add and multiply #'s by #'s: 5 boxes \times 10 widgets = 50.
 - We can multiply \$'s by #'s: 100 * \$3 = \$300.
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Category theory is the accounting system for coherent structures.

- It makes analogies—similarities of structure—into formal objects.
- It's been useful in math, CS, physics, materials science, linguistics, etc
- What sort of system accounts for PCE3?

My understanding of PCE3

Again, please excuse my ignorance of PCE3; here's how I naively see it.

- Our concept of the early earth is that life formed from chemistry.
- We see a repeated pattern regarding concentration of power.
- X's *power* is its capacity to quickly catalyze a change in conditions.
 - In physics, it is work divided by time. But more colloquially also:
 - Birthing a child is a 9-month catalysis: $ygote + food \rightarrow baby$.
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- To catalyze and not be dissipated during those reactions is "survival".
- We're interested in a formal account of power.
 - By what mechanism does it so consistently concentrate?
 - Is there a theme to the sorts of changes that life catalyzes?
 - In what respects are material conditions on the EE relevant?

Concentration of power comes from organization, not elementary makeup.

- Spider silk is stronger—pound for pound—than steel.
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Any life-form is a collective, a dynamic organization of smaller parts.

- The organization provides an interaction pattern for the parts.
- The RNA will interact with the nucleus and the ribosome, etc.
- What occurs during these interactions can change the organization.
- As an extreme example, death will allow the system to disintegrate.
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- ...according to some pattern: the type of signals/materials that flow.
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- ...according to some pattern: the type of signals/materials that flow.
- The interaction pattern itself can change based on what flows.
- I have a category-theoretic accounting system for such things.

Plan for the talk

During the remainder of the talk I will:

- discuss open dynamical systems and interaction patterns,
- explain dynamic organizational structures, and
- conclude with a summary.

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I Introduction

2 Dynamical systems and interaction patterns

- Open dynamical systems
- Interaction patterns

3 Dynamic organizational structures

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Open dynamical systems

An open dynamical system (ODS) is a kind of "worker".

- It can interact with the outside world through a specified interface.
- This interface displays its position, attitude, various signals, and...
- ...can receive forces, materials, signals, etc.
- The ODS has an internal state, which dictates its position, etc., and...
- ...this state can change based on the forces, etc., that it receives.

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We can consider either continuous or discrete ODS's.

- In both cases, the outputs are given by a function of the state.
- In a cont'ous ODS, the state evolves according to a system of ODEs...
- ...and the inputs provide time-varying parameters to the ODEs.
- In a discrete ODS, the state is updated discretely based on input.

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How can you visualize the interface of an ODS (continuous or discrete)?

- The interface may change through time, but at any point it can...
- ...be drawn as a box with inputs/outputs on the left/right: ↓

Wiring diagrams and interaction patterns

Open dynamical systems connect with each other in interaction patterns.

The easiest sort of interaction pattern to visualize is a wiring diagram.



- This one interconnects interfaces p_1, \ldots, p_5 inside a boundary q.
- Let's denote the wiring diagram as $\varphi \colon (p_1, \ldots, p_5) \to q$.
- It gives rise to a formula: given an ODS in each p_i , we get one in q.

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■ In particular, WDs are implemented in software (AlgebraicDynamics.jl) Interaction patterns can be more general, e.g. weighted, dynamic, prob'istic.

It's harder to draw, but the unambiguous CT syntax generalizes easily.

Nesting

Interaction patterns can be nested.



Here we see eight interfaces and three interaction patterns. Check!

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You can abstract an interacting population of ODS's as an ODS.

The semantics of zooming or regrouping is compositional wrt nesting.

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3 Dynamic organizational structures

- Mode dependent interaction
- Continuity with respect to boundary
- Applications to PCE3

4 Conclusion

Mode dependent interaction

In the above pictures, the interaction pattern appeared to be fixed.

- This works great for integrated circuits, but not for real life.
- Even computers have "plug and play": the wiring pattern can change.
- We're talking now, but later we'll disconnect.
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Moreover, the interface of each ODS can change through time.

- Sometimes your eyes are open, and your input type is larger.
- Sometimes your car is in a tunnel and can't receive GPS.
- The system's interface depends on its mode: "mode dependence".

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The CT accounting system I've been alluding to can handle all this.

- The formal system is called polynomial functors; it accounts for:...
- ...ODS's in time-varying interfaces with time-varying inter'n patterns.

A dynamic organizational structure¹ is a systematized version of the above.

- It's like a presentation of the laws of nature or the rules of a game.
- It tells you rules for how parts are allowed to form into wholes...
- ...where these laws/rules are required to be a compositional.

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- Hebbian learning (spike-time dependent plasticity),
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In each case: a systematized dynamic for how parts form wholes.

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More detail on dynamic organizational structures

Recall this picture, where parts form wholes.



Several things are different for a dynamic organizational structure.

- The interaction pattern is generalized, e.g. weighted sums.
- The inter'n pattern is dynamic, changing based on what flows.
- The dynamic inter'n pattern is formulaic and compositional.

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- The inter'n pattern is dynamic, changing based on what flows.
- The dynamic inter'n pattern is formulaic and compositional.
- This "formulaic and compositional" aspect shows up in our examples.
 - Deep learning formula: gradient descent.

• A population of gradient descenders forms a gradient descender.

- Prediction markets formula: update wealths based on Bayes rule.
 - A population of predictors with varying wealths forms a predictor.

Applications to PCE3

In Darwinian evolution, the individual is metaphysically primary.

- People thus need to argue about whether groups can be selected for.
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- The theoretician needs to be able to change boundary lines.
- Low-latency feedback loops may better define individuality...
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The above gives an accounting system for ODS's inhabiting interfaces.

- It is compositional: continuous with respect to boundary-drawing.
- Individuality is a derived concept rather than metaphysically primary.
- Parts connect or disconnect based on what occurs during interaction.

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- **2** Dynamical systems and interaction patterns
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Only four known interesting examples, but PCE3 may offer another one.

- We'd need to find a system for how identity-flexible evolution works.
- The math gives guardrails (compositionality) that ensure integrity.
- Email david@topos.institute if you'd like to explore a collaboration.

Thanks! Comments and questions welcome...